Indian Institute of Information Technology, Allahabad

Neural Networks

By

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Department of Information Technology
Indian Institute of Information Technology, Allahabad

TEAM

Computer Vision and Biometrics Lab (CVBL)

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Indian Institute of Information Technology Allahabad

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WE HAVE LEARNED SO FOR IN THIS MODULE

Image features and categorization

Choosing right features

Object, Scene, Action, etc.

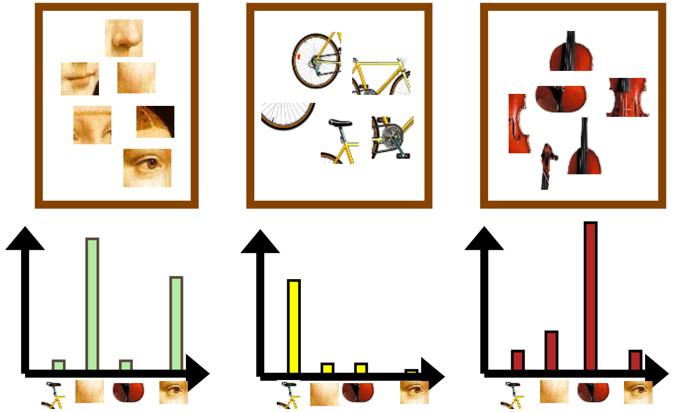
Bag-of-visual-words

Extract local features

Learn "visual vocabulary"

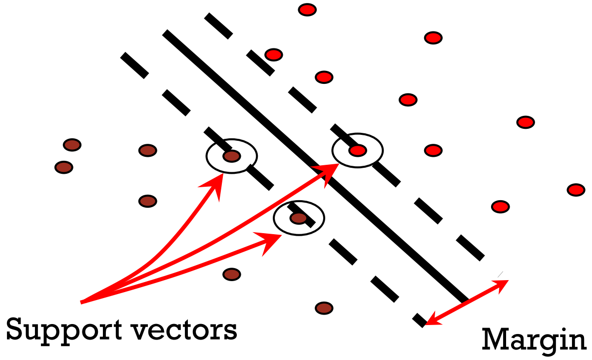
Quantize features using visual vocabulary

Represent by frequencies of "visual words"



Classifiers

Nearest neighbor, KNN, Linear classifier, SVM, Non-linear SVM, Multi-class SVM, Softmax classifier



TODAY'S CLASS

Optimization

Gradient Descent & Back propagation

Update rule

Neural networks

OPTIMIZATION

Optimization is the process of finding the set of parameters W that minimize the loss function.

Strategy #1:First very bad idea solution: Random search:

Simply try out many different random weights and keep track of what works best.

Strategy #2: Random local search:

Start out with a random W, generate random changes δW to it and if the loss at the changed $W+\delta W$ is lower, we will perform an update.

Strategy #3: Following the gradients:

There is no need to randomly search for a good direction: this direction is related to the gradient of the loss function.

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GRADIENT DESCENT

The procedure of repeatedly evaluating the gradient of loss function and then performing a parameter update.

Vanilla (Original) Gradient Descent:

```
while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Mini-batch Gradient Descent (MGD):

```
while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Stochastic Gradient Descent (SGD):

Special case of MGD when mini-batch contains only a single example

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INTERPRETATION OF THE GRADIENT

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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$$f(x,y) = x + y \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = \qquad \frac{\partial f}{\partial y} =$$

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$$f(x,y) = \max(x,y) \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = \qquad \qquad \frac{\partial f}{\partial y} =$$

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$$f(x,y) = \max(x,y) \quad \rightarrow \quad \frac{\partial f}{\partial x} = 1(x >= y) \quad \frac{\partial f}{\partial y} = 1(y >= x)$$

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COMPOUND EXPRESSIONS WITH CHAIN RULE

$$f(x, y, z) = (x + y)z$$

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$$f(x, y, z) = (x + y)z$$

↓

$q = x + y$ and $f = qz$

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Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

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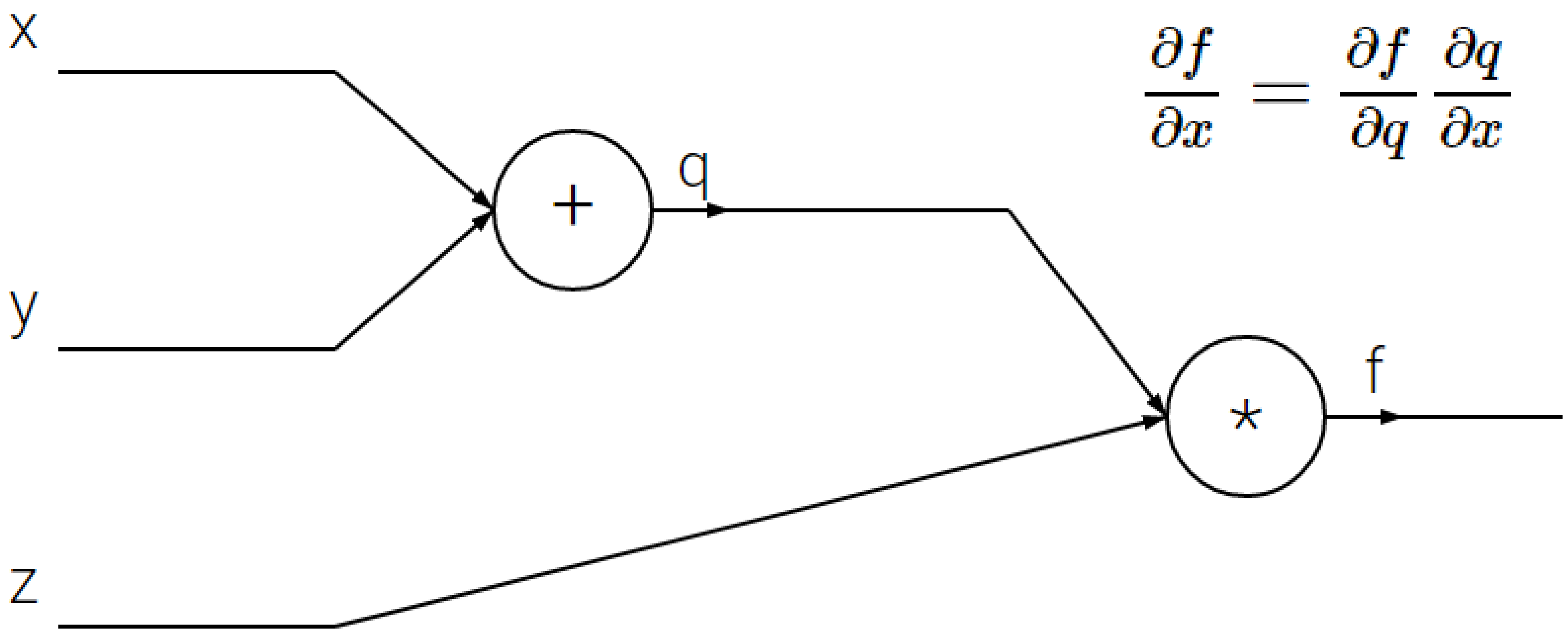

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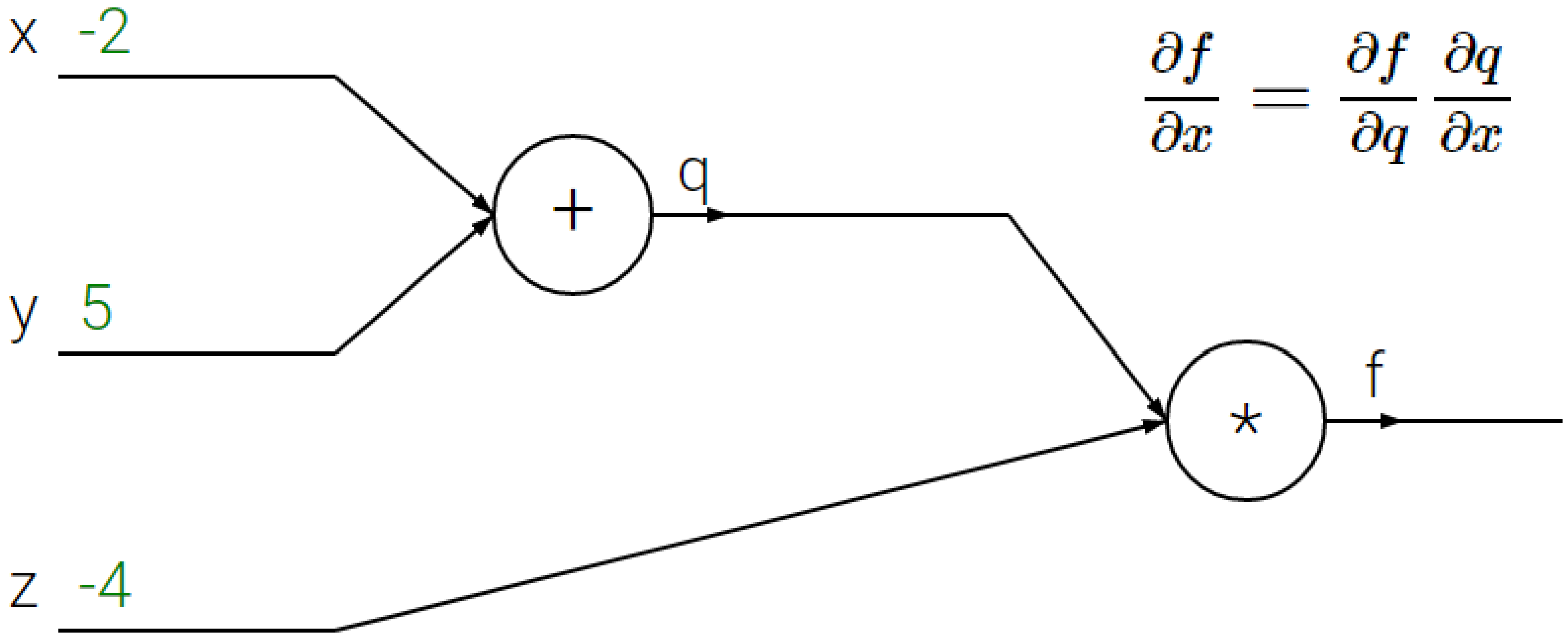

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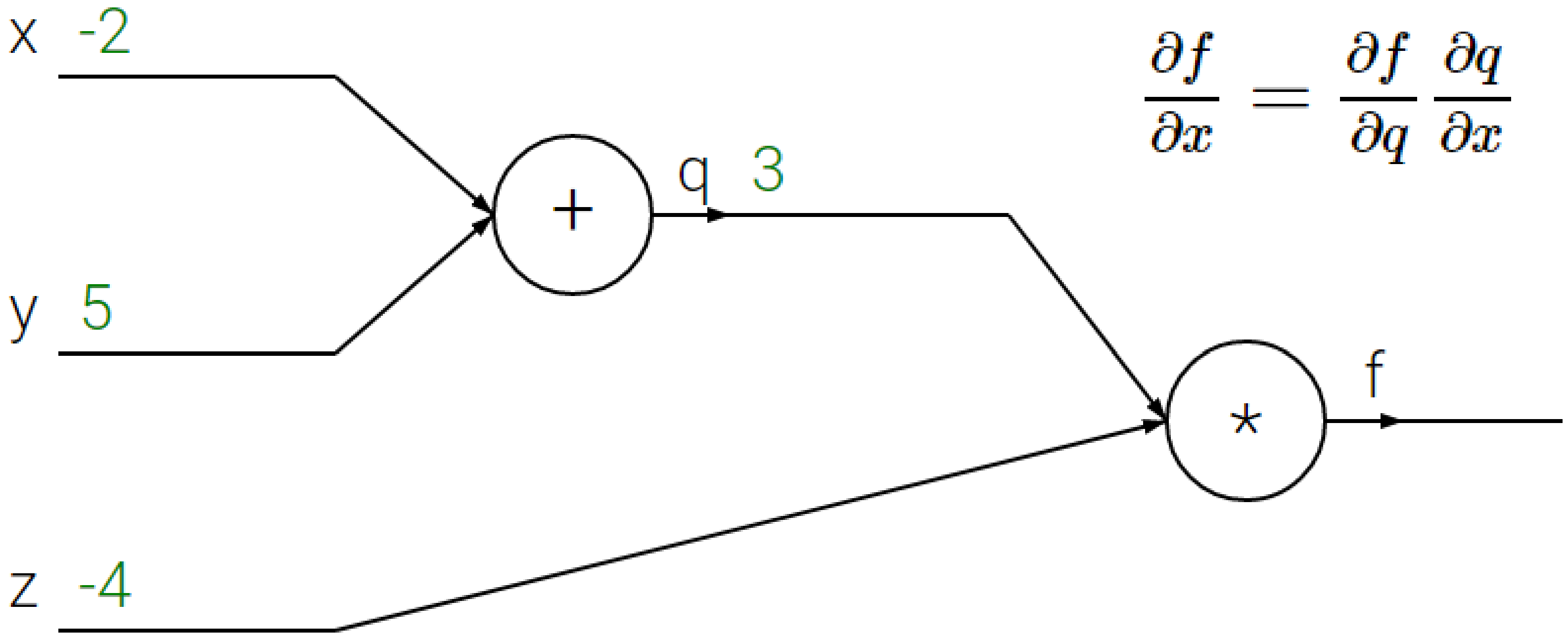

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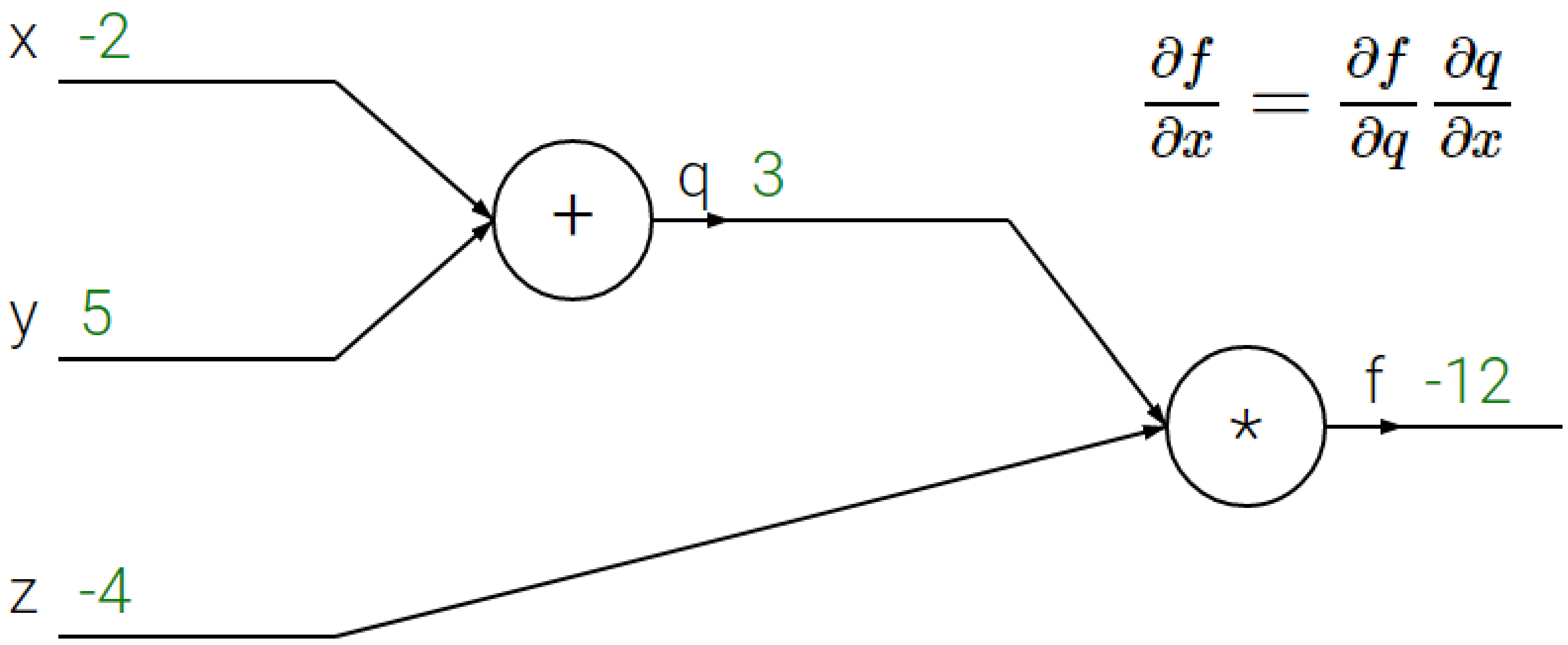

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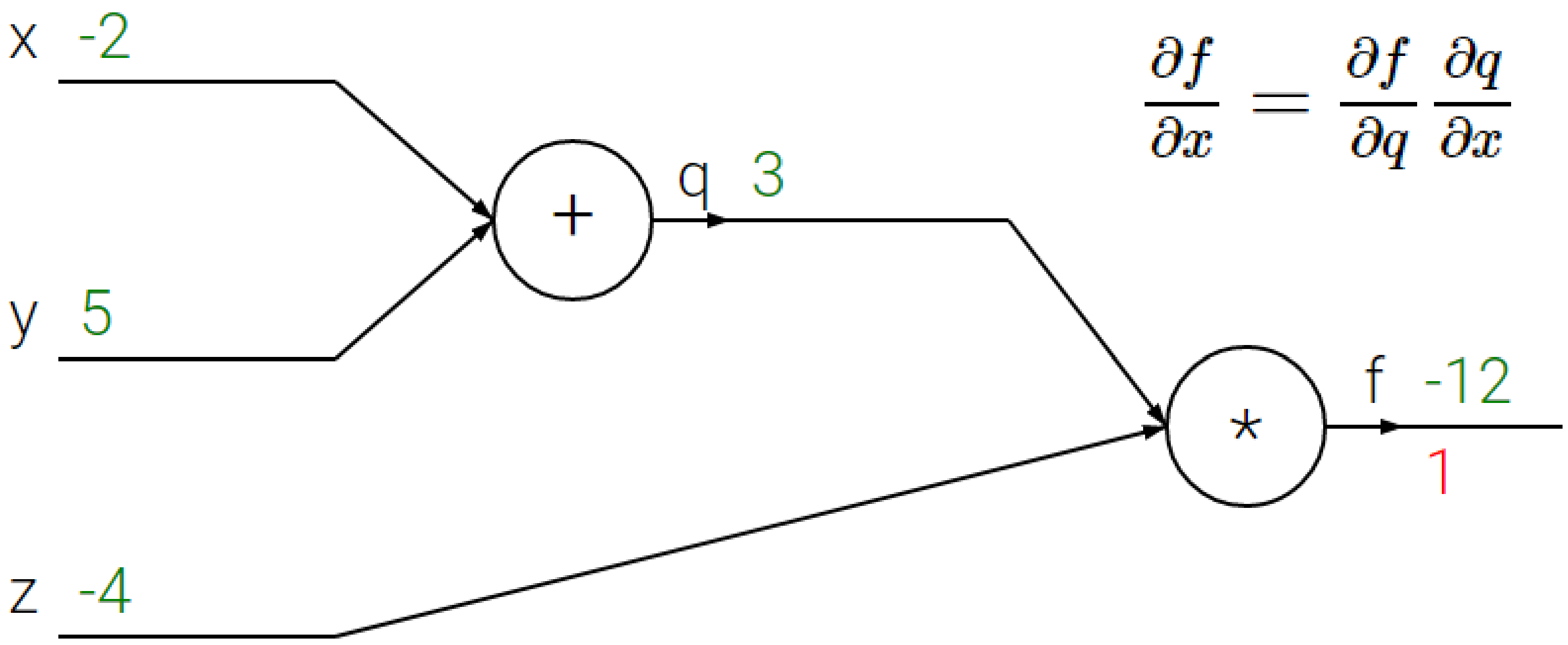

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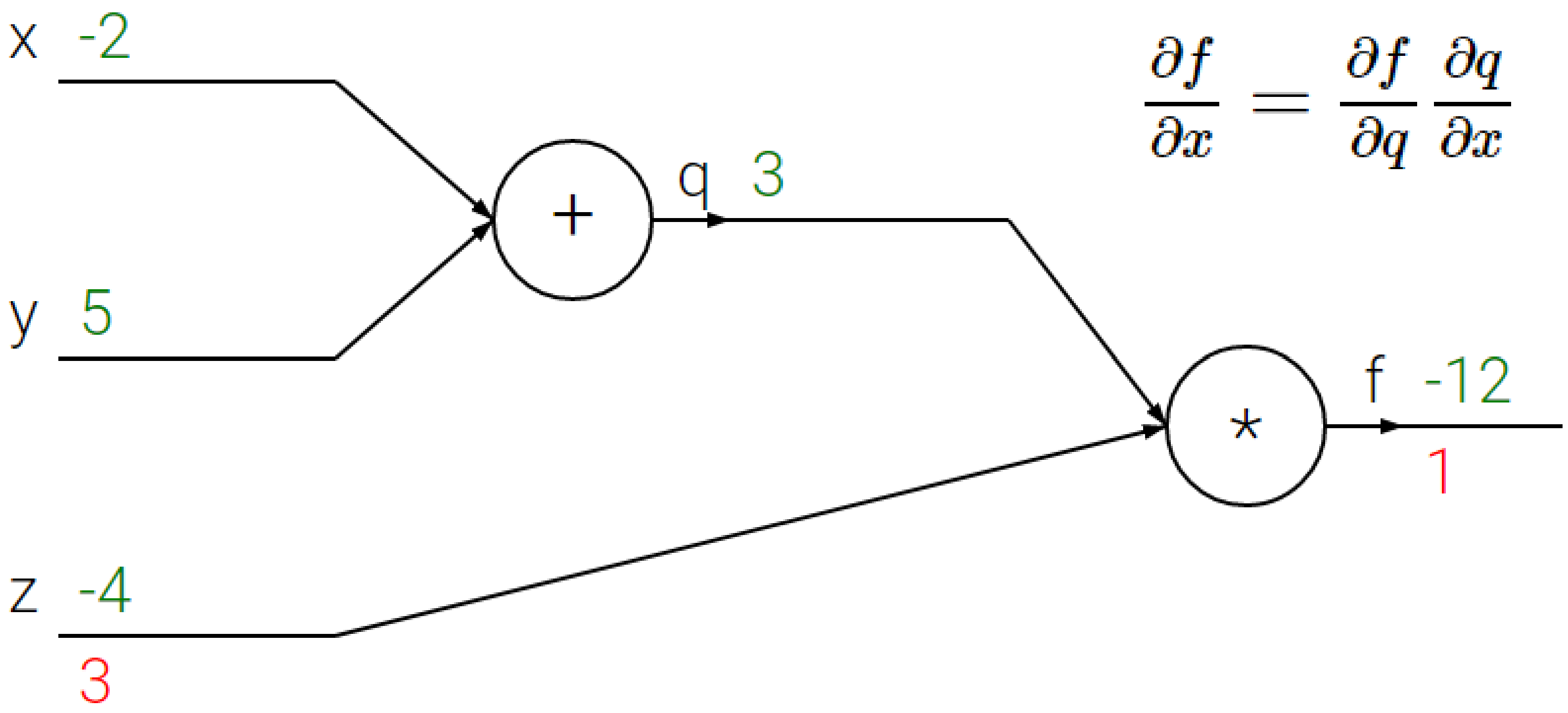

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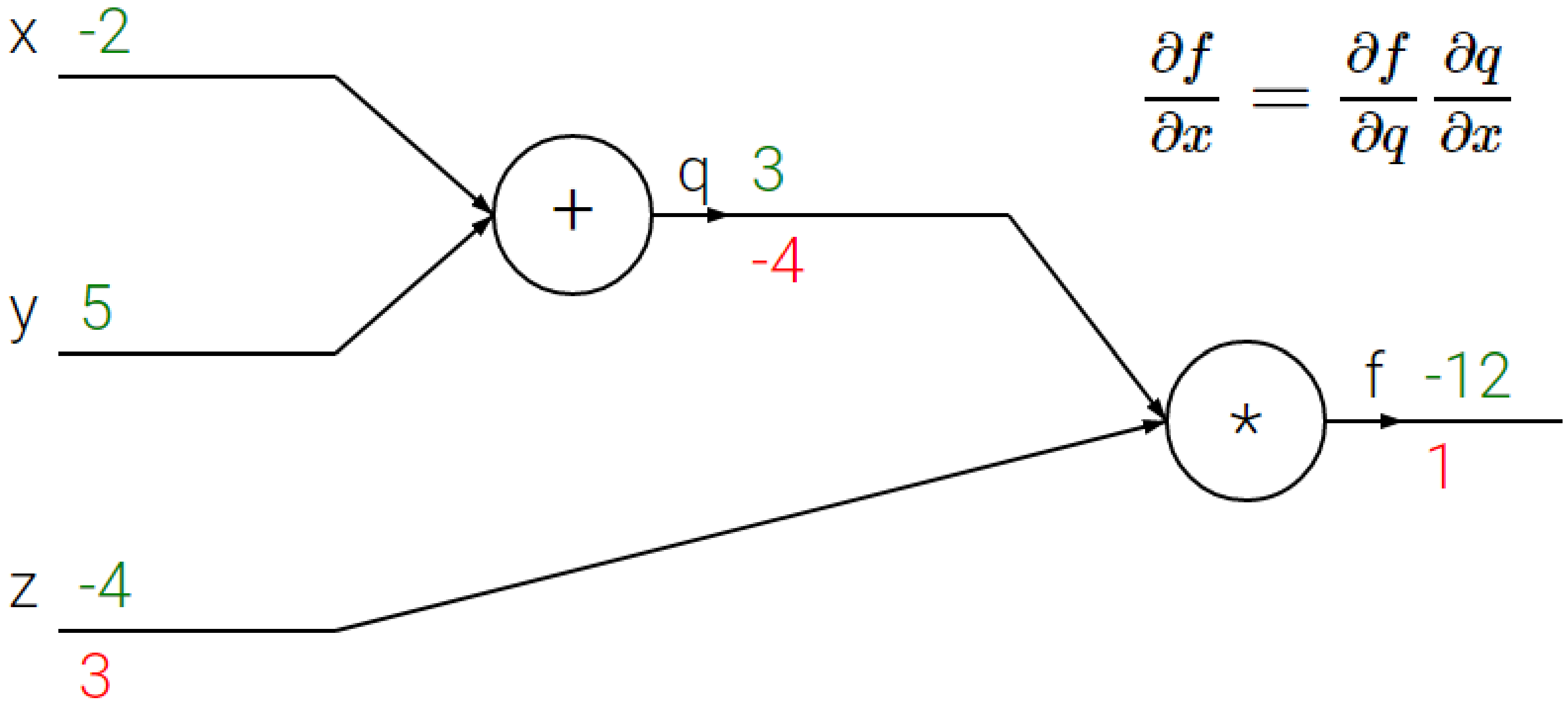

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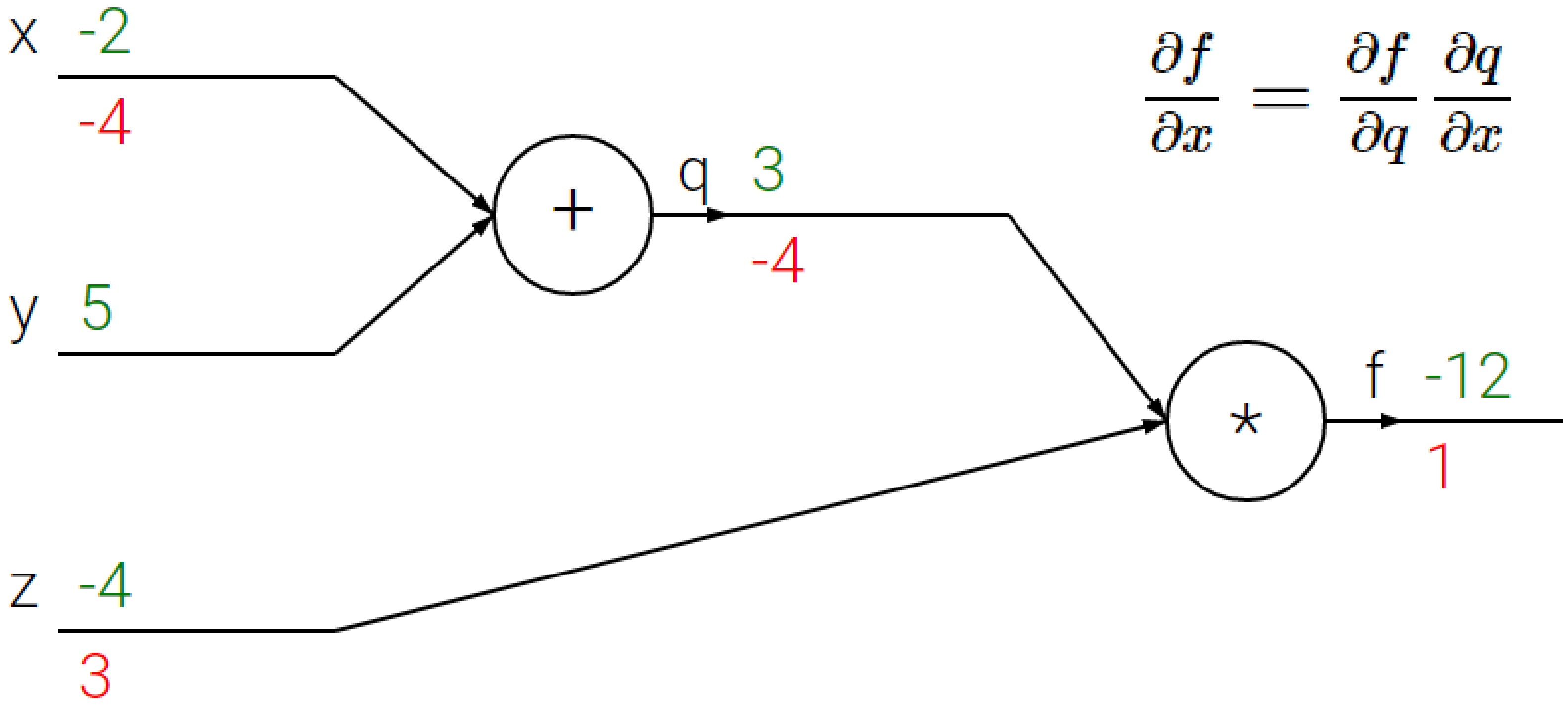

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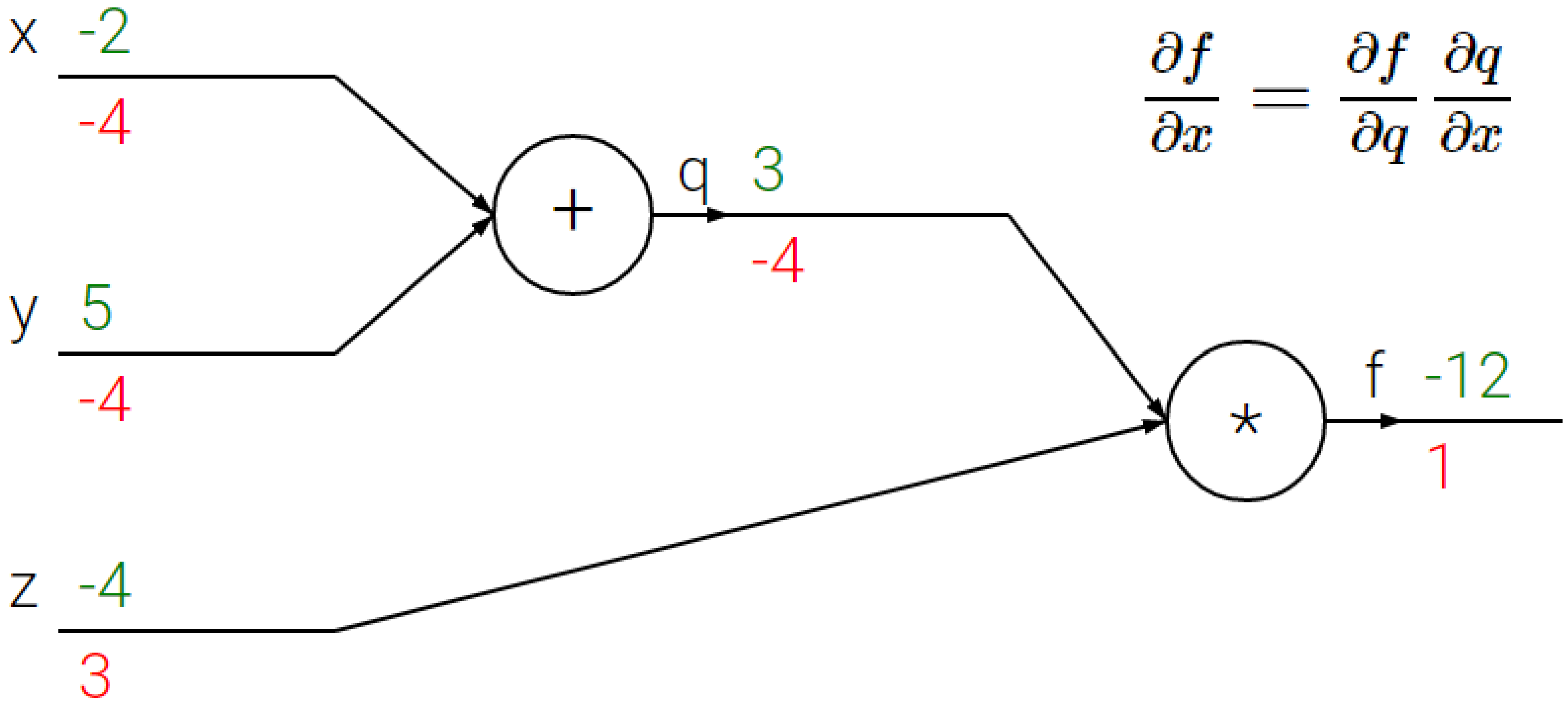

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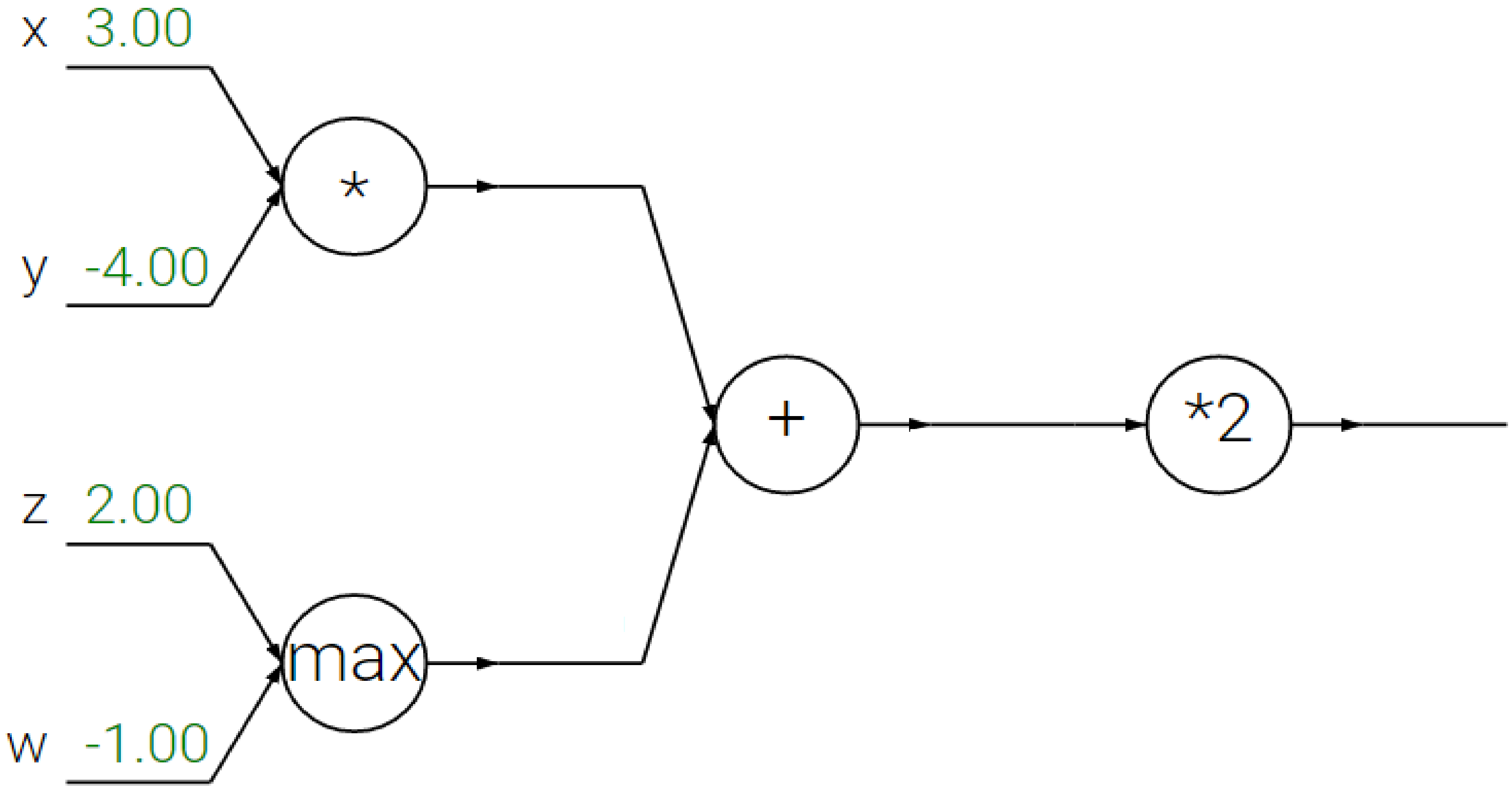
FORWARD AND BACKWARD PASS



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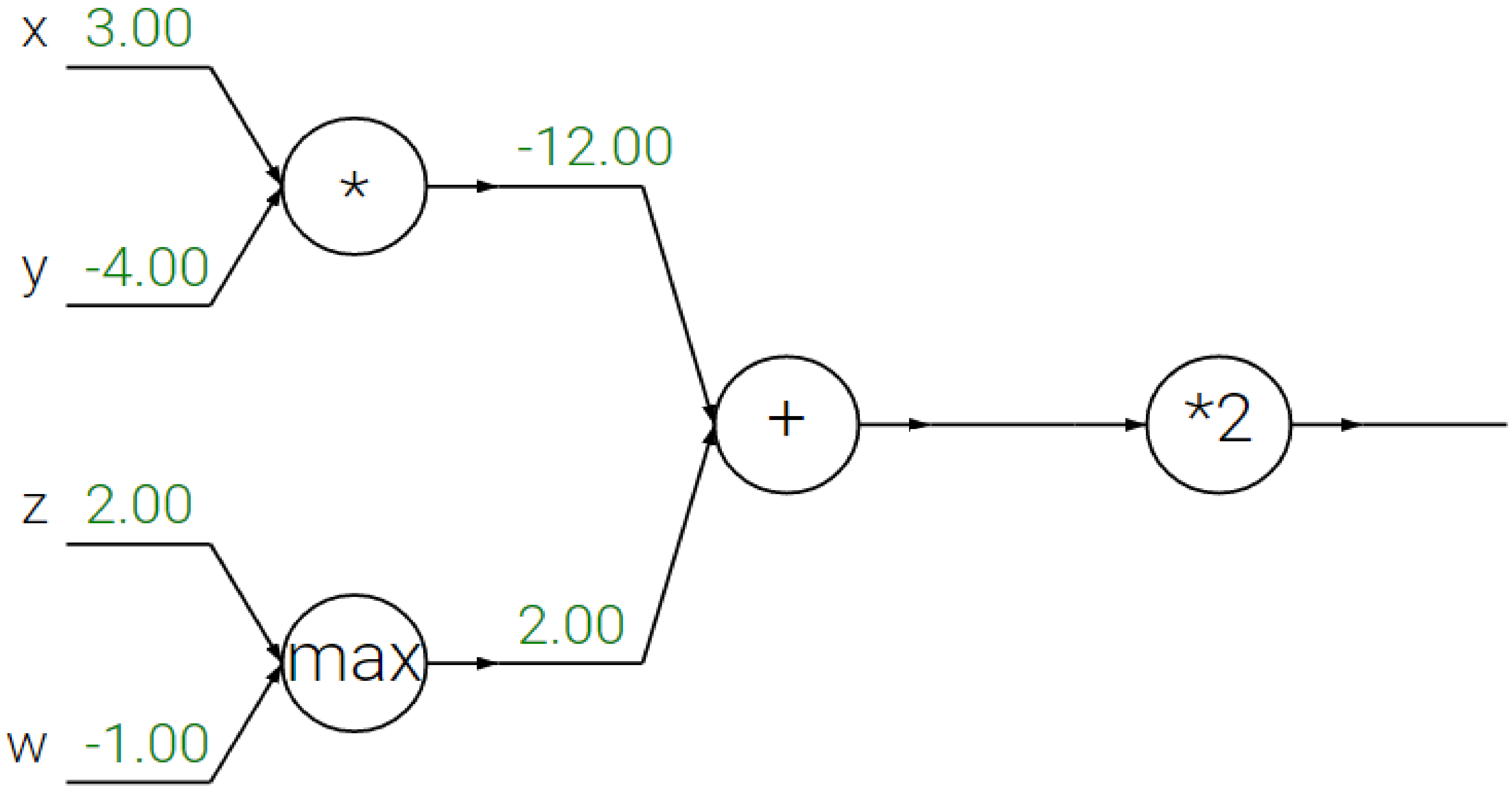

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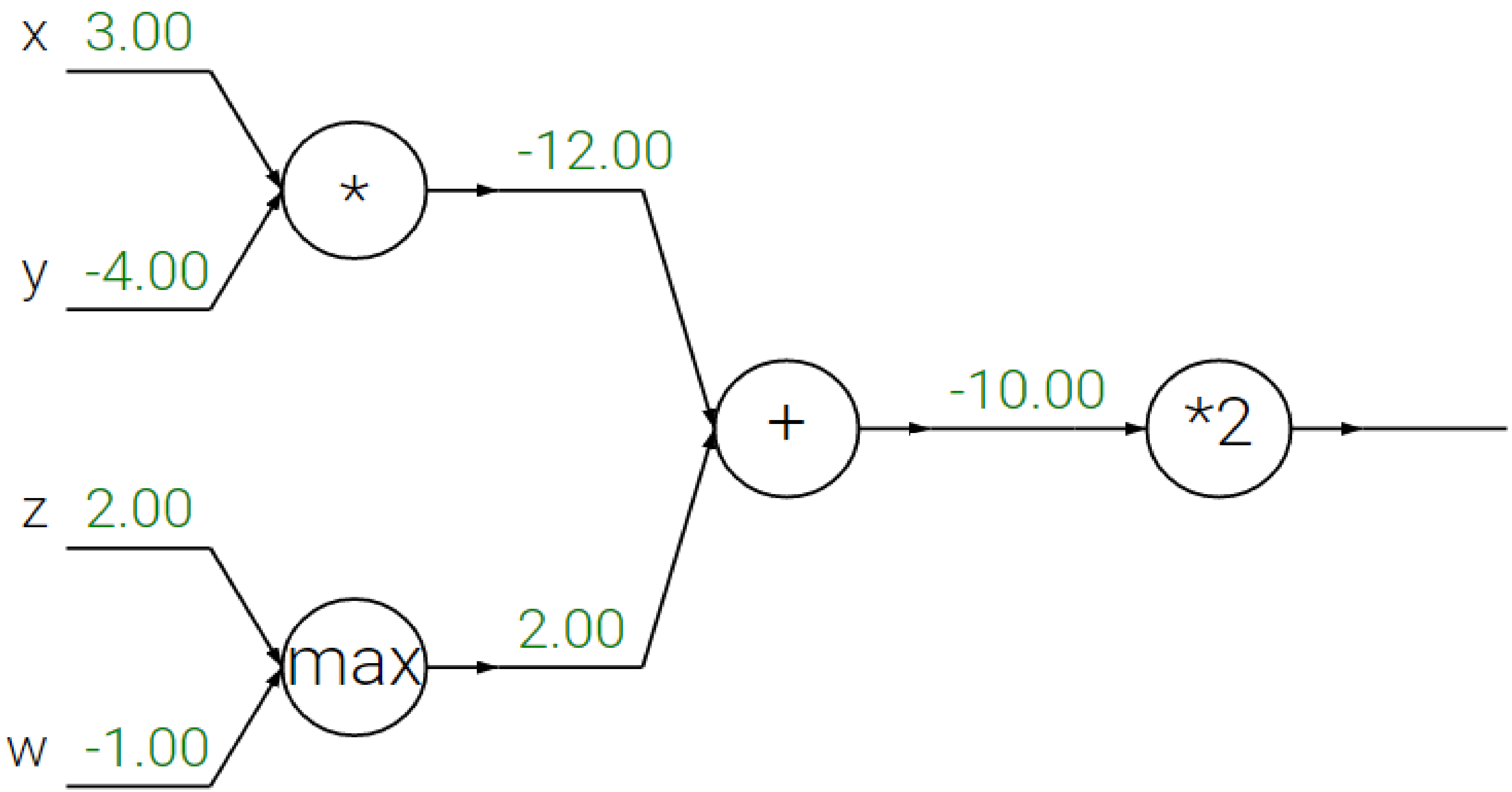

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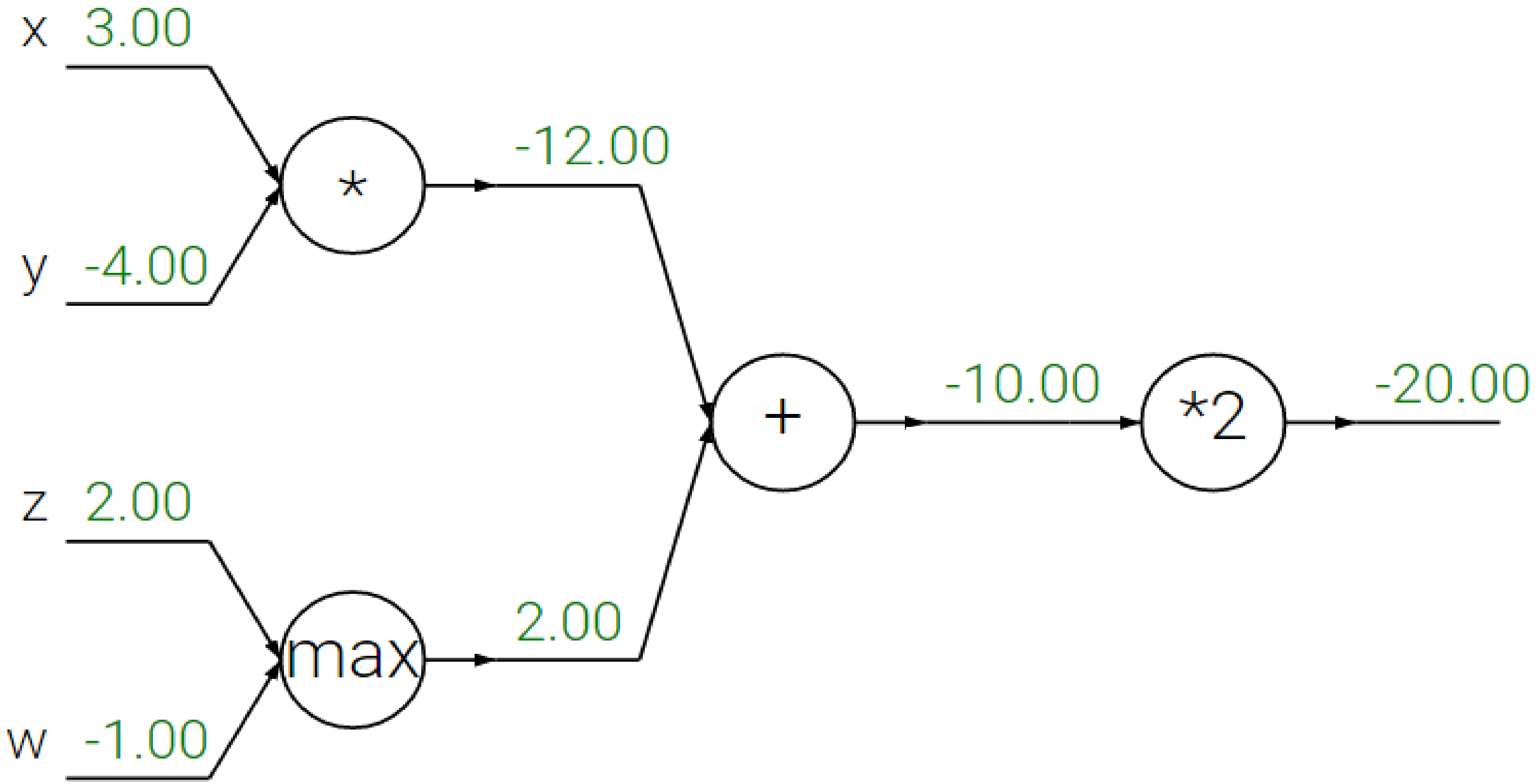

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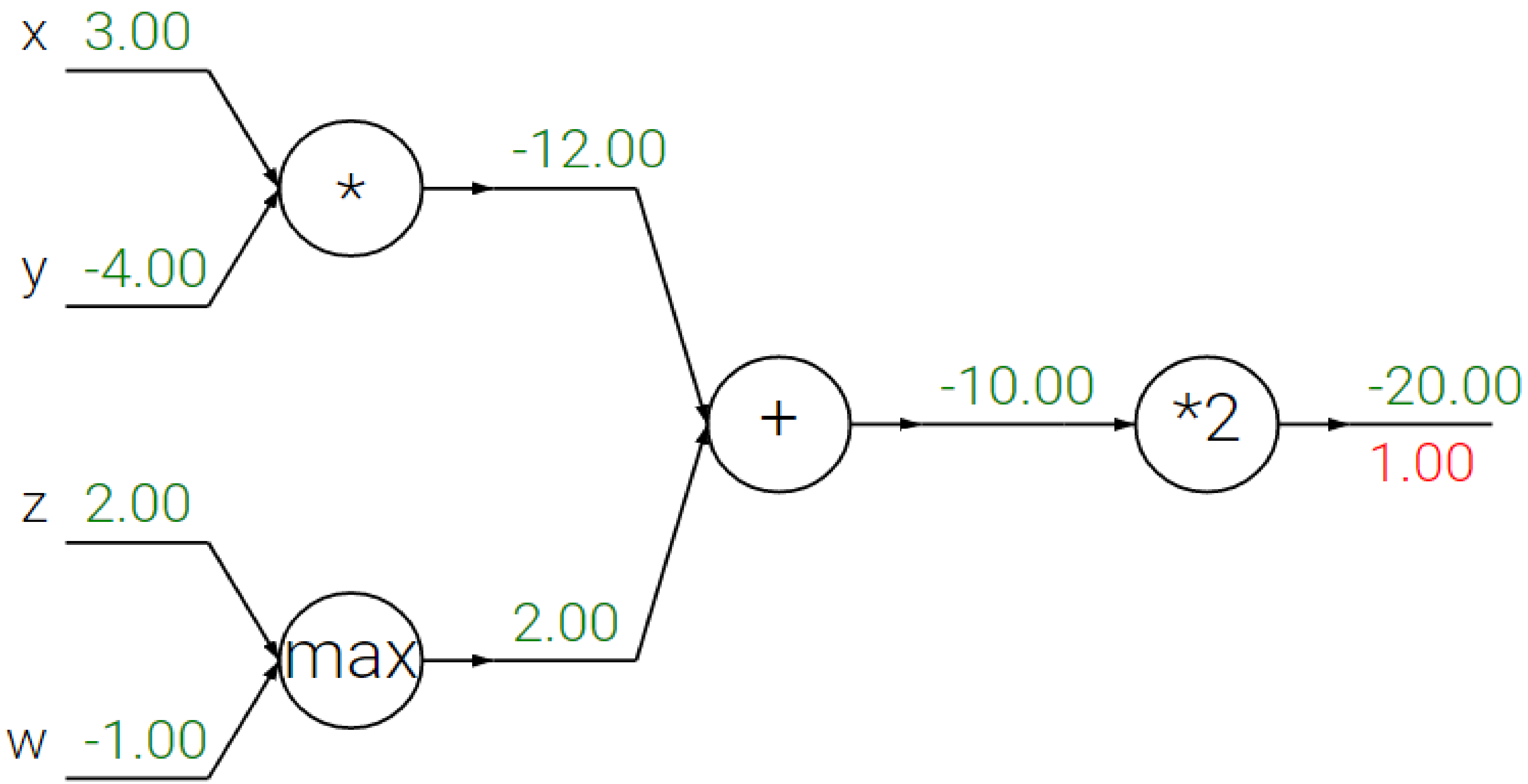

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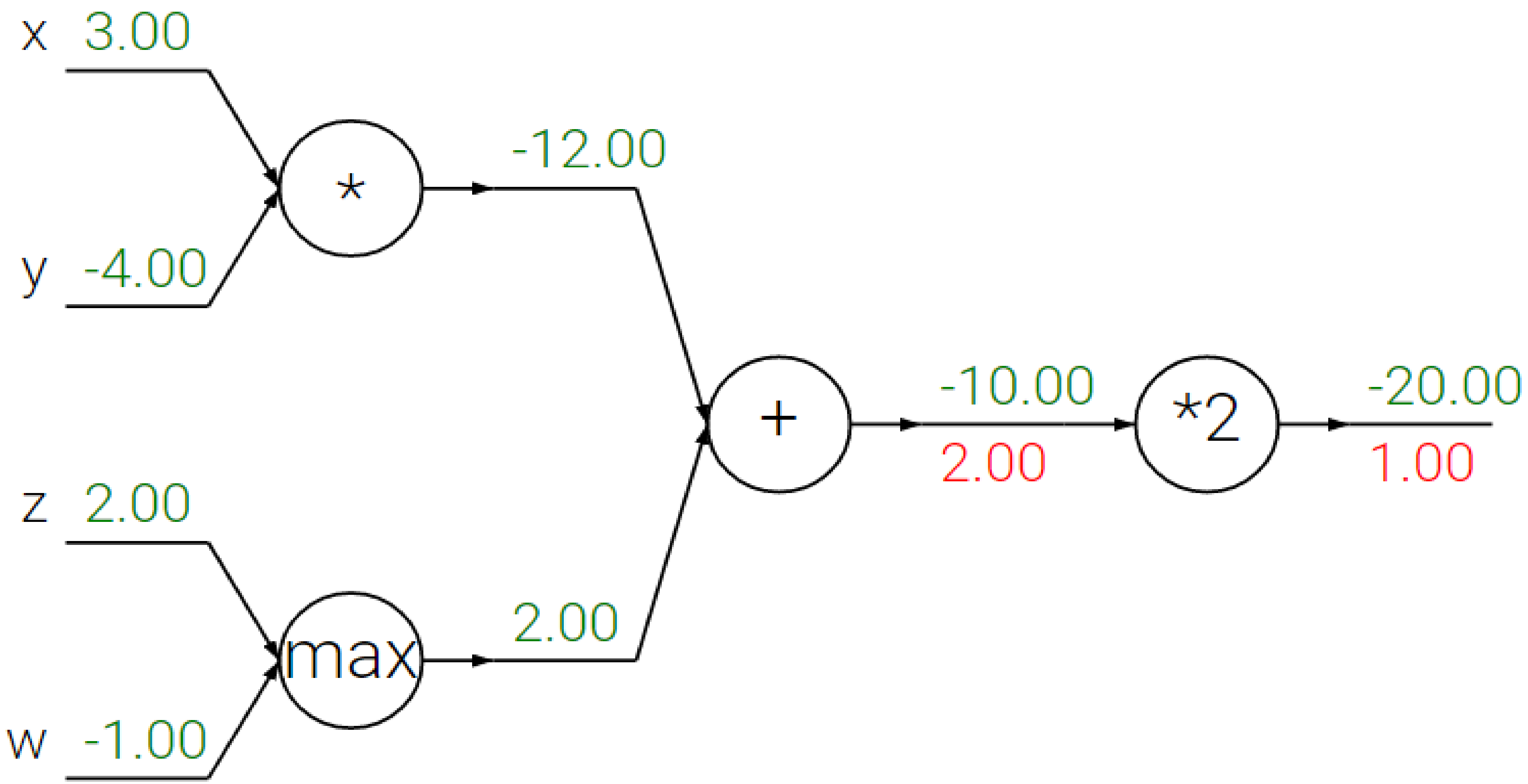

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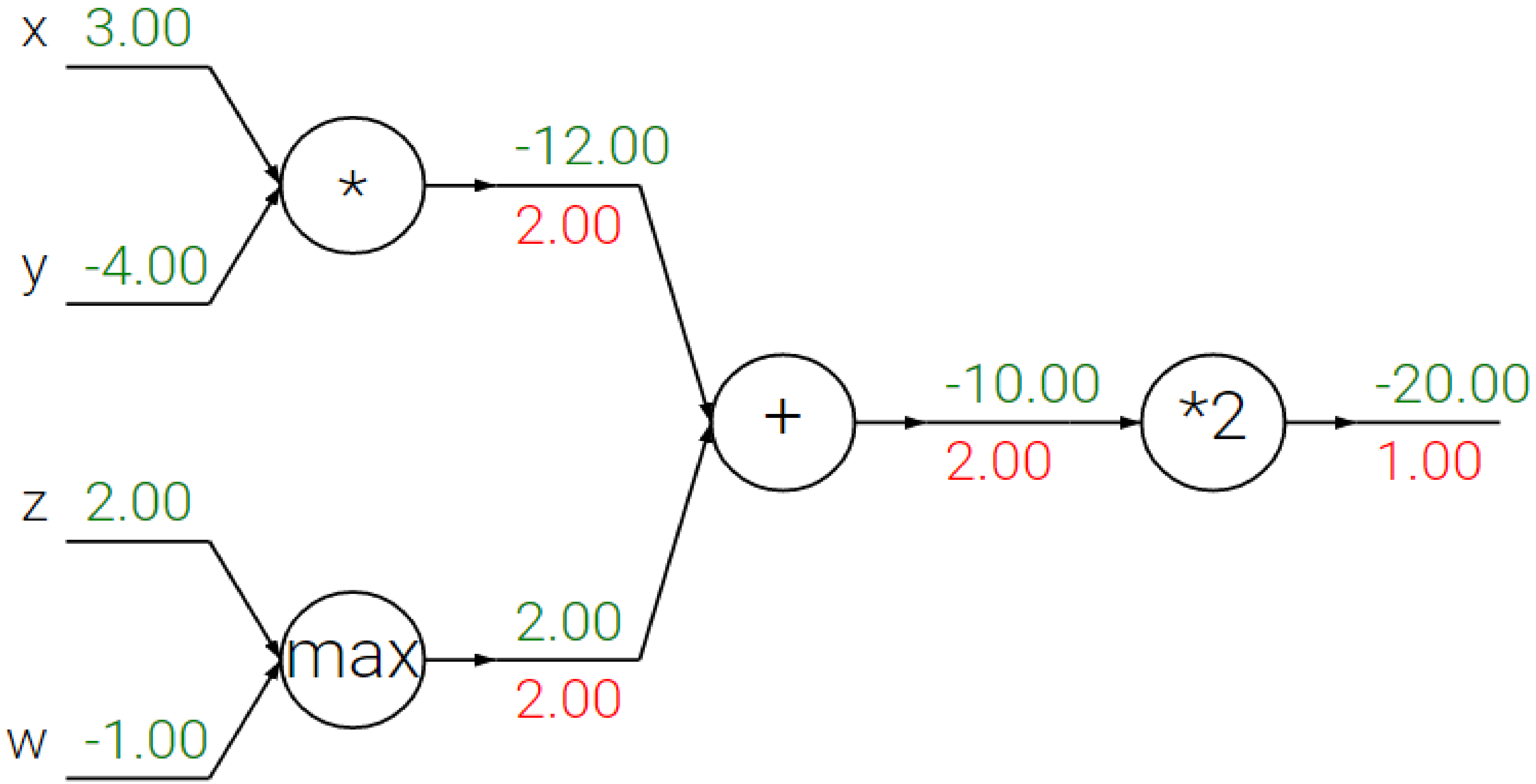

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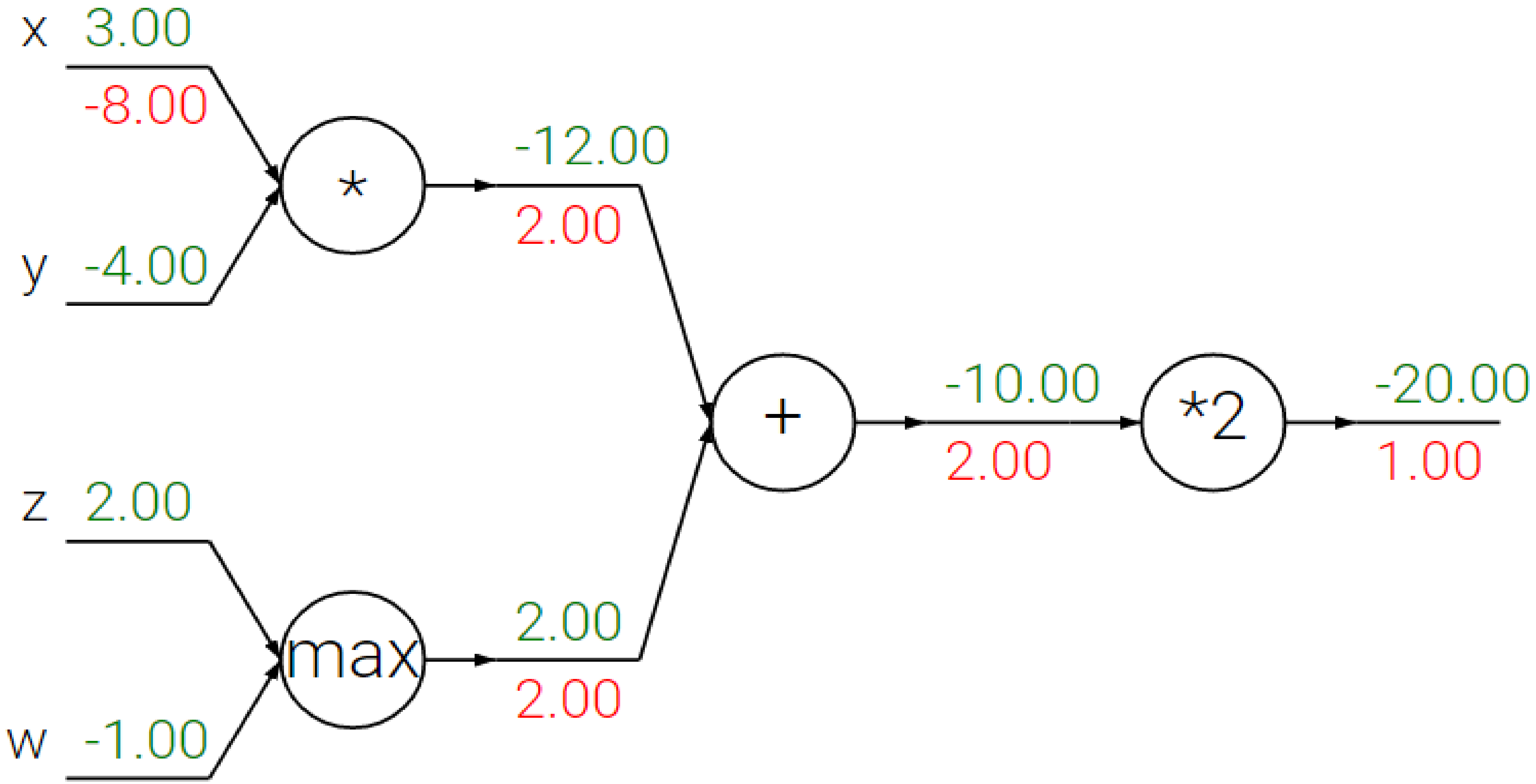

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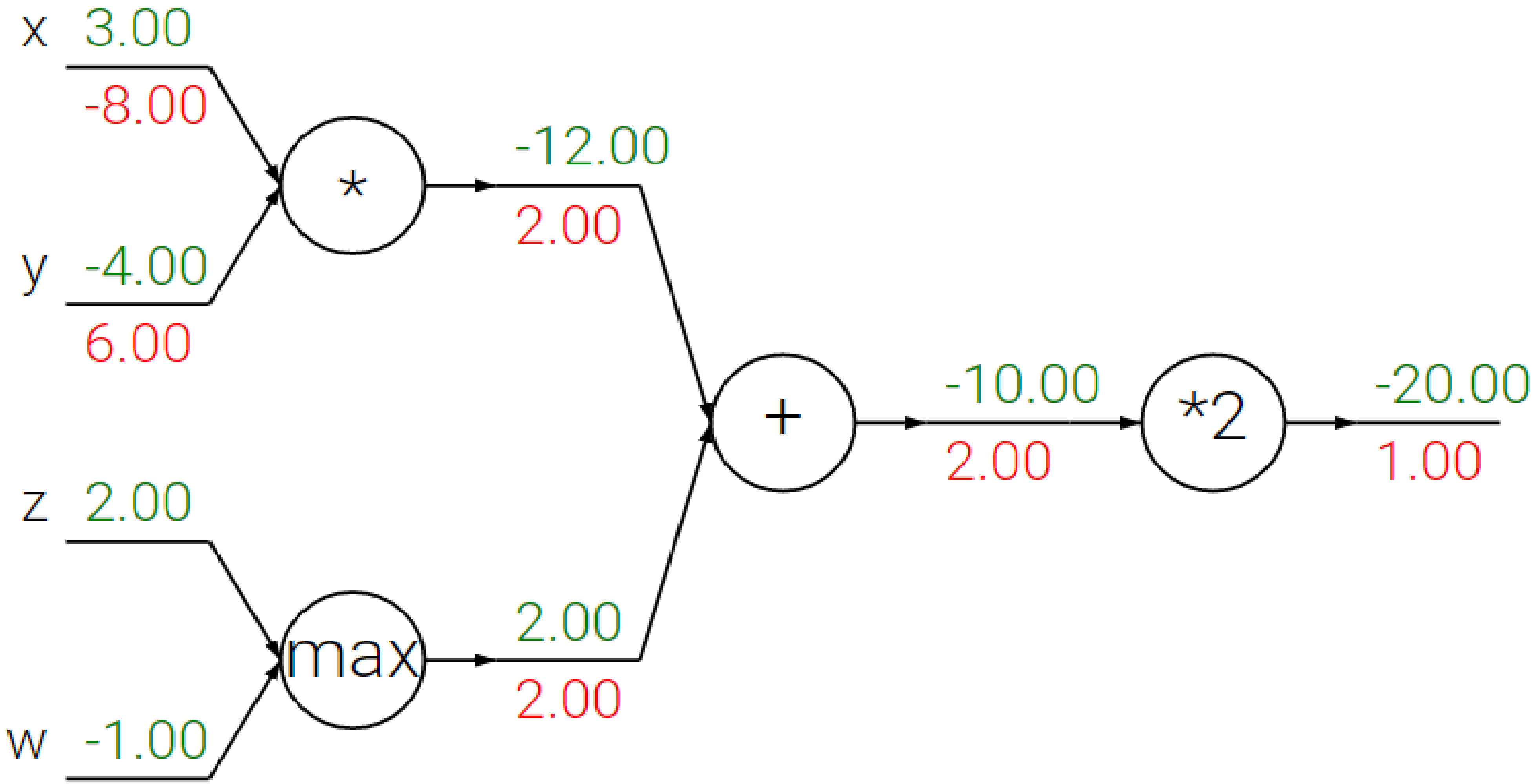

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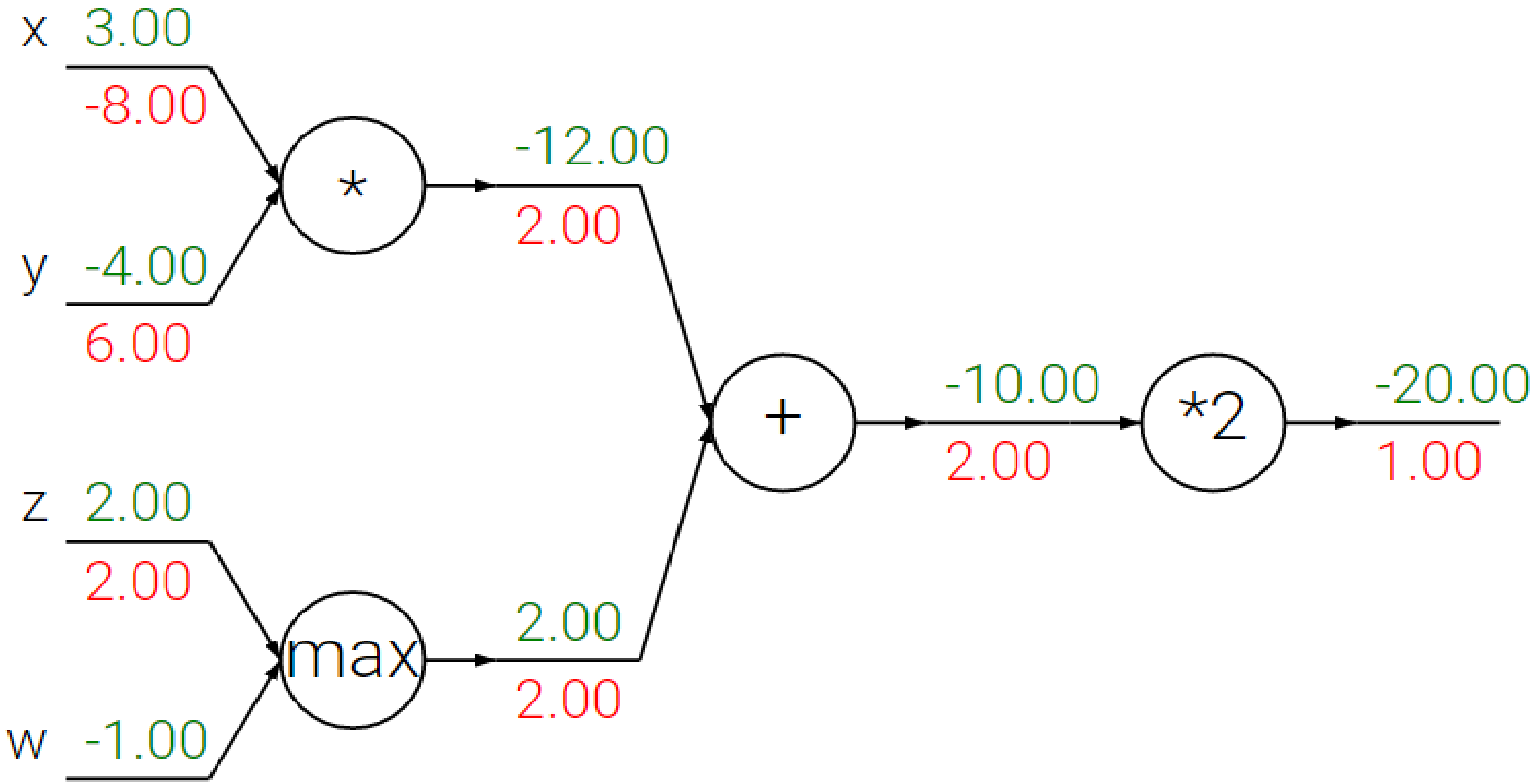

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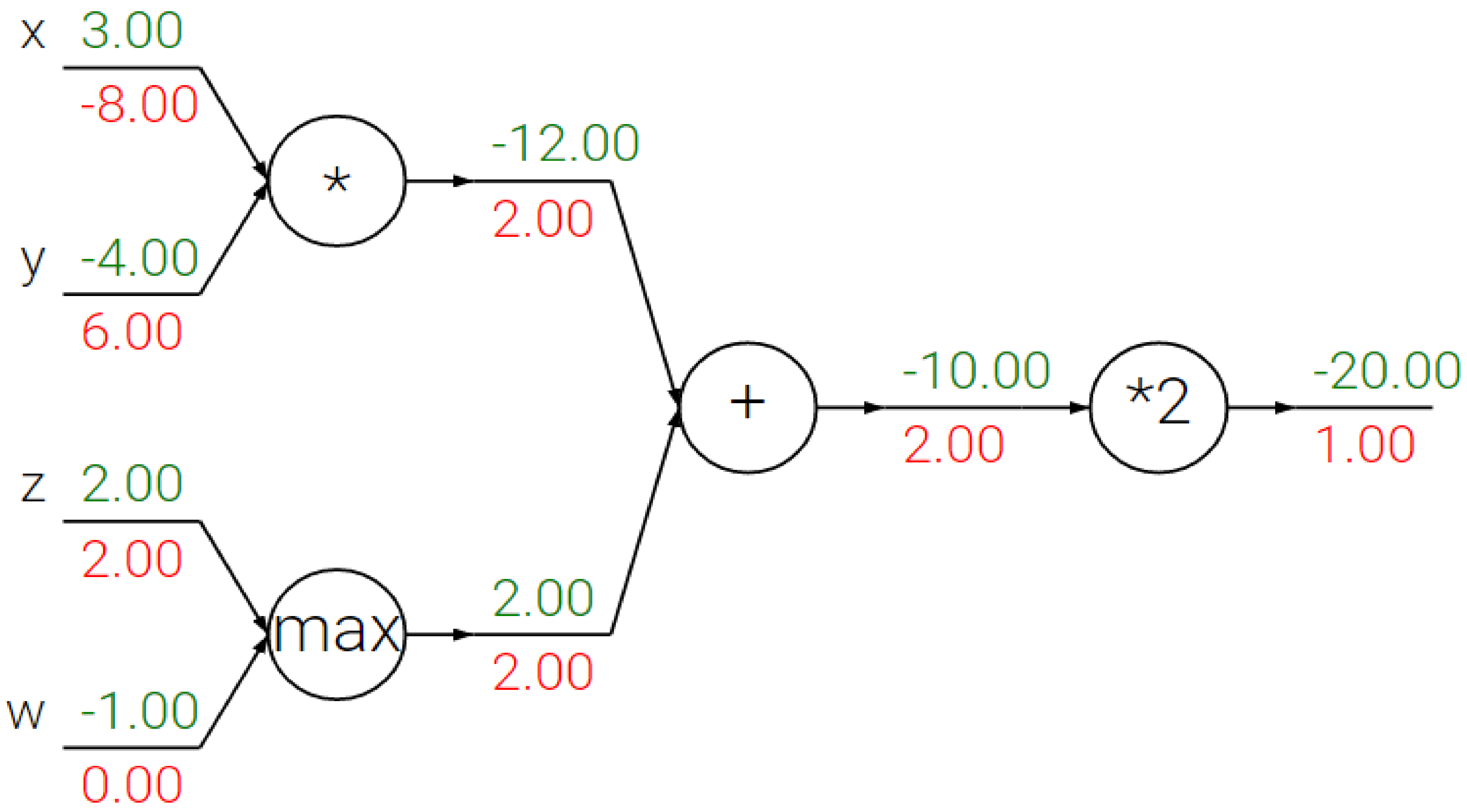

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SIGMOID EXAMPLE

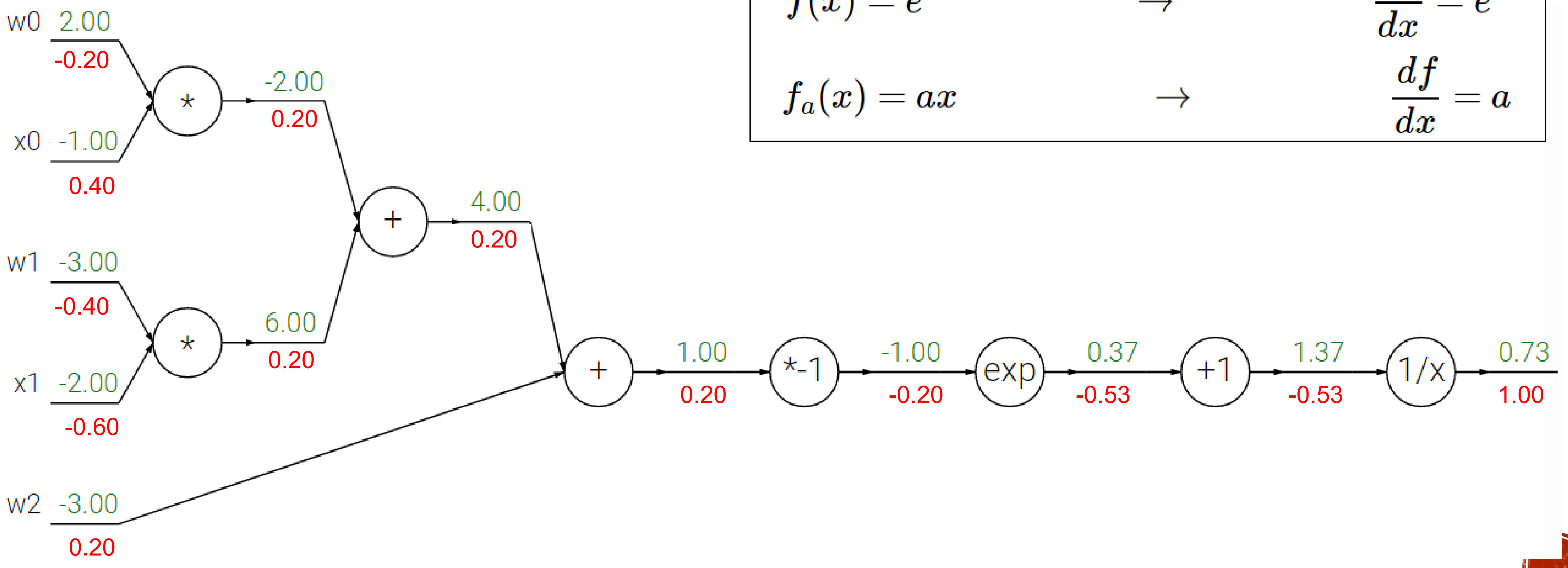
$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$



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SVM LOSS: GRADIENT

SVM loss function for a single datapoint (without regularization):

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

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$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

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Gradient w.r.t. w_{y_i} :

$$\nabla_{w_{y_i}} L_i = -\left(\sum_{j \neq y_i} 1(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right) x_i$$

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Count of the number of classes that didn't meet the desired margin

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$$\nabla_{w_{y_i}} L_i = -\left(\sum_{j \neq y_i} \mathbb{1}(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right) x_i$$

Count of the number of classes that didn't meet the desired margin

Gradient for the other rows where $j \neq y_i$:

$$\nabla_{w_j} L_i = \mathbb{1}(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) x_i$$

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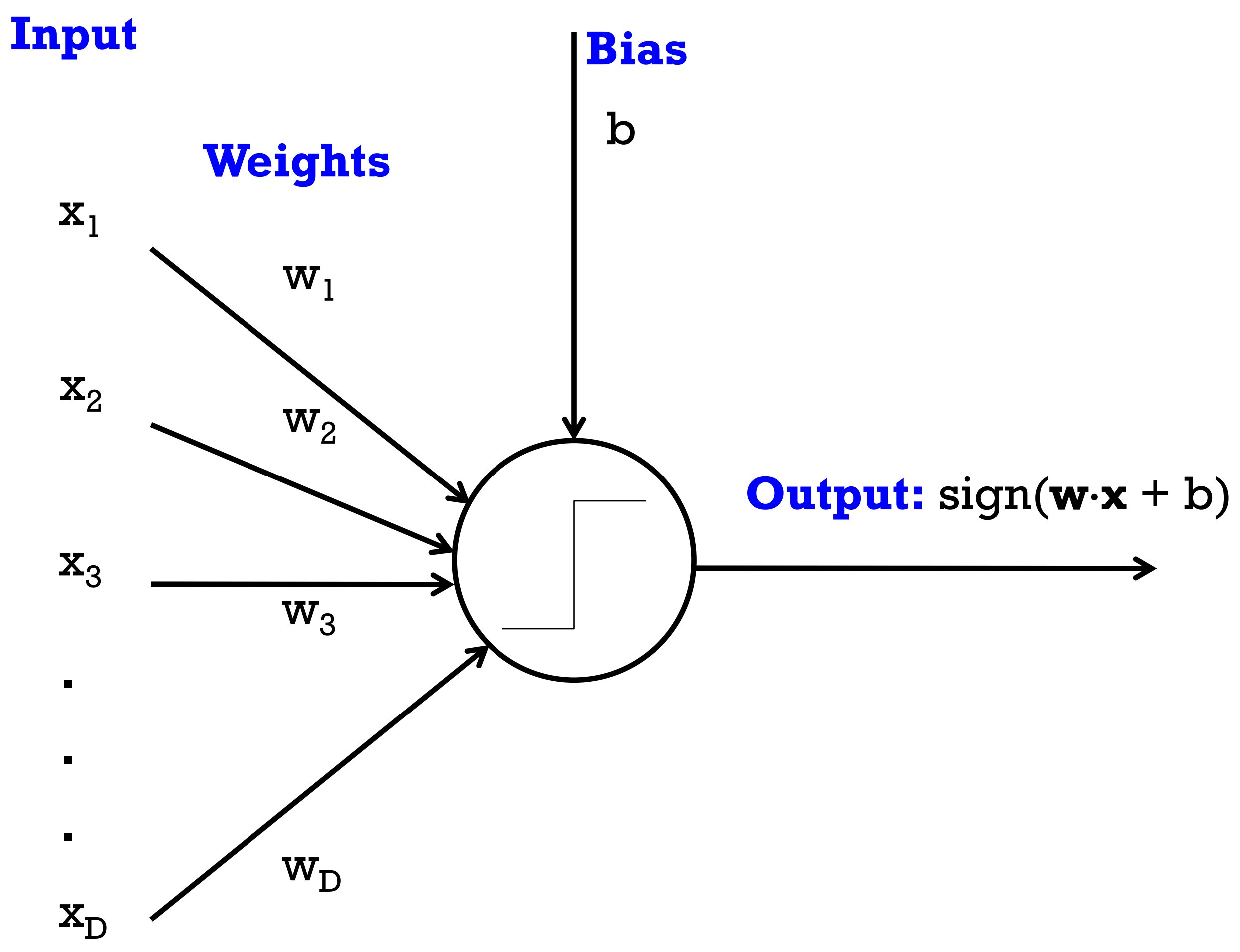
PERCEPTRON

- Supervised learning of binary classifier



SINGLE NEURON AS A LINEAR CLASSIFIER

Binary Softmax classifier (*Logistic Regression*) $\sigma(\sum_i w_i x_i + b)$

Source: http://cs231n.github.io

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Binary Softmax classifier (*Logistic Regression*) $\sigma(\sum_i w_i x_i + b)$

↓

Probability of one of the classes: $P(y_i = 1 \mid x_i; w)$

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SINGLE NEURON AS A LINEAR CLASSIFIER

Binary Softmax classifier (*Logistic Regression*) $\sigma(\sum_i w_i x_i + b)$

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Probability of the other class would be:

$$P(y_i = 0 \mid x_i; w) = 1 - P(y_i = 1 \mid x_i; w)$$

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Binary SVM classifier:
Alternatively, we could attach a max-margin hinge loss to the output of the neuron and train it to become a binary Support Vector Machine.

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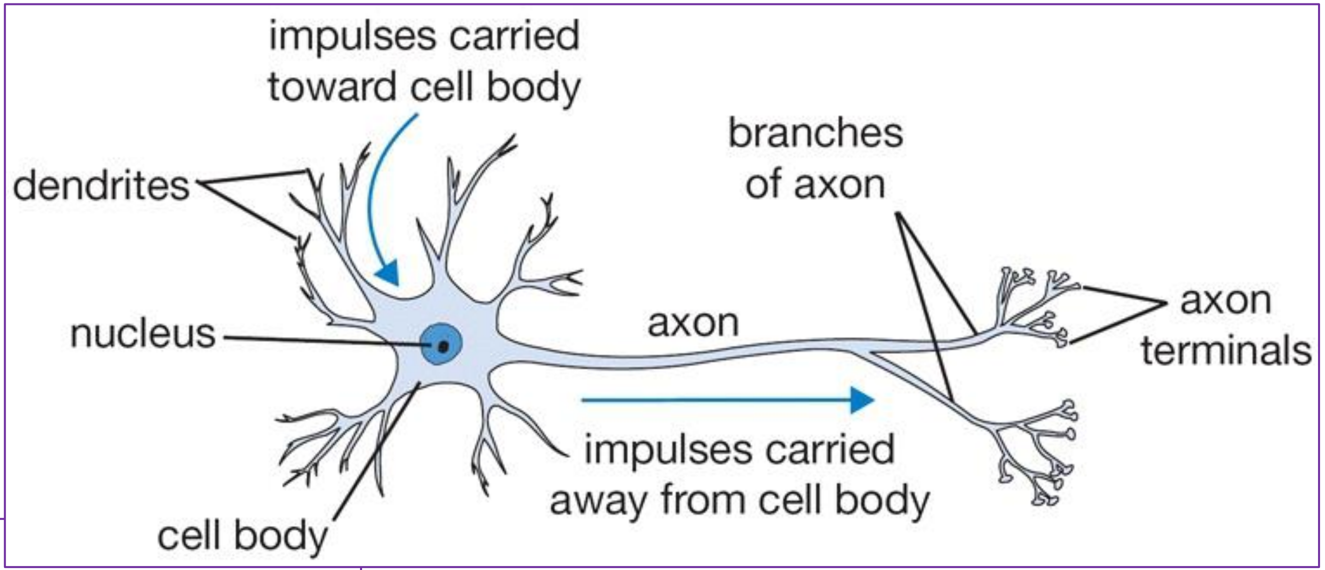
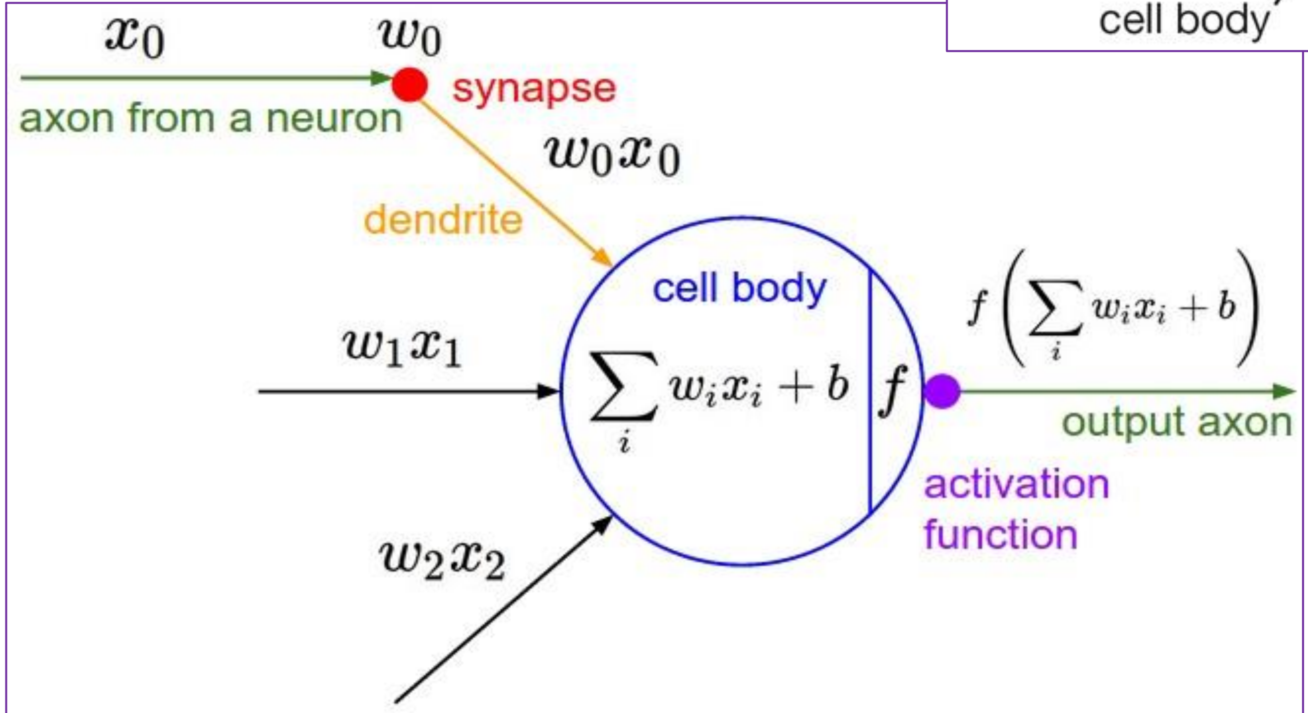
LOOSE INSPIRATION: HUMAN NEURONS

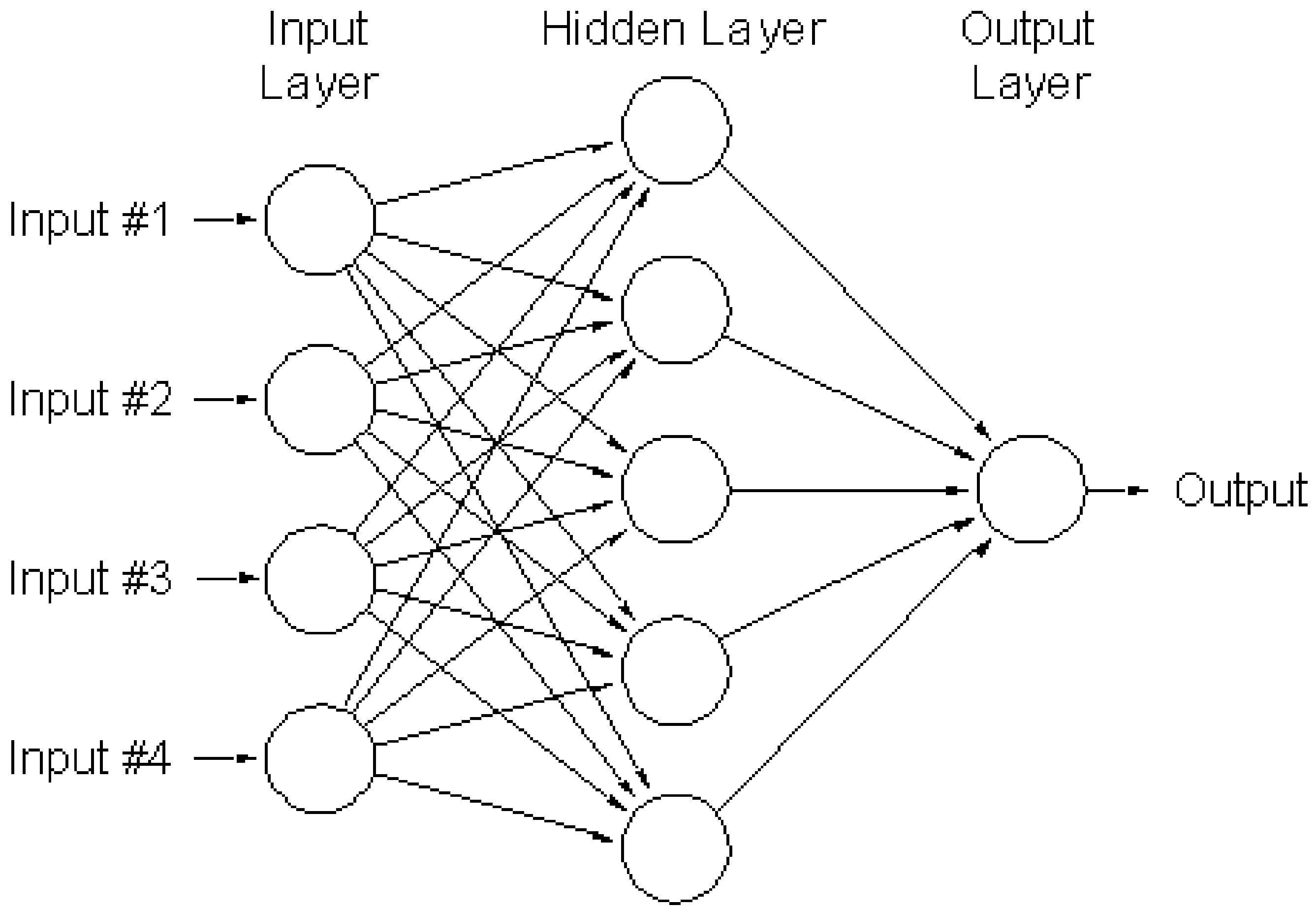




Source: http://cs231n.github.io

MULTI-LAYER NEURAL NETWORKS

- Network with a hidden layer:



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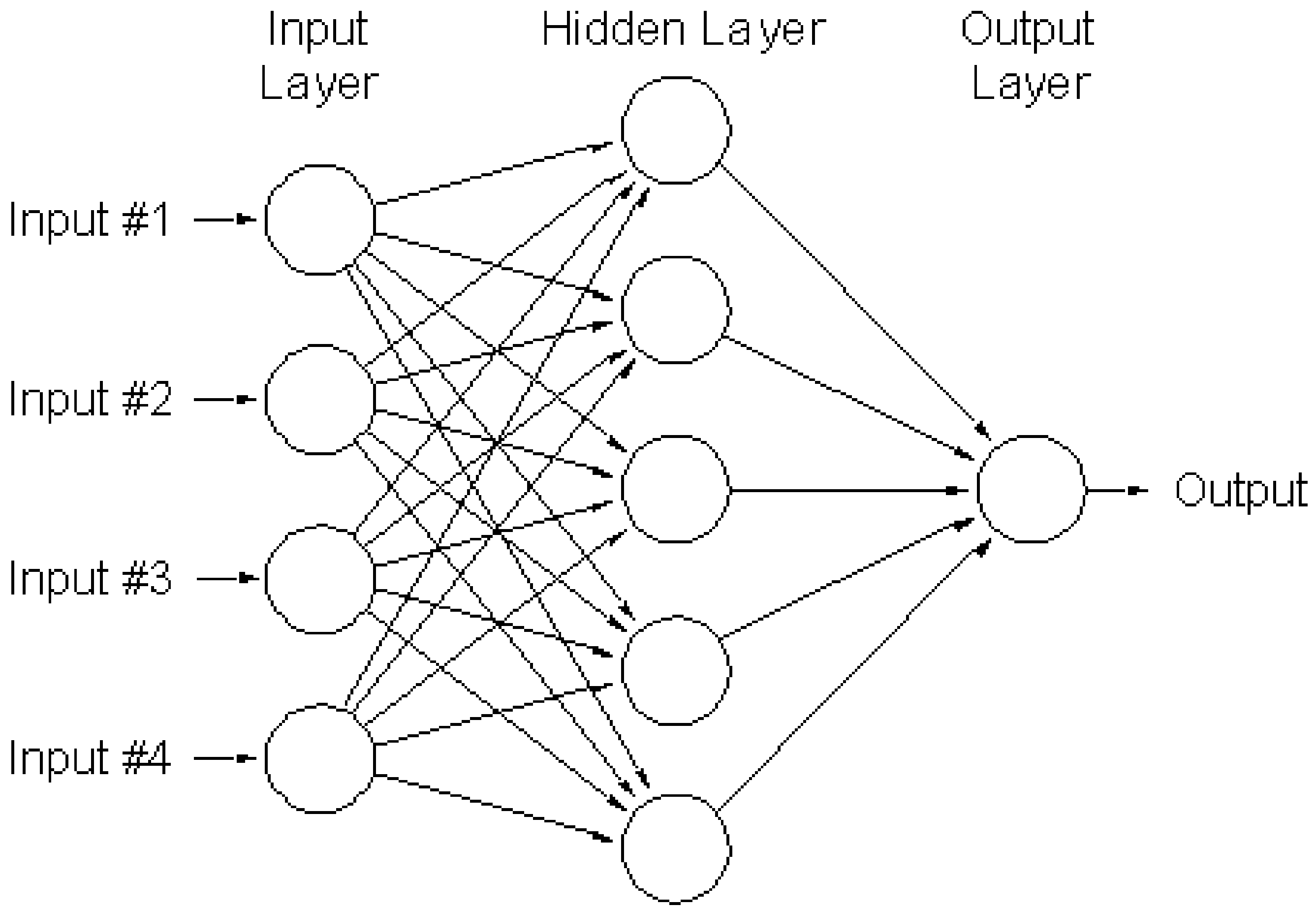
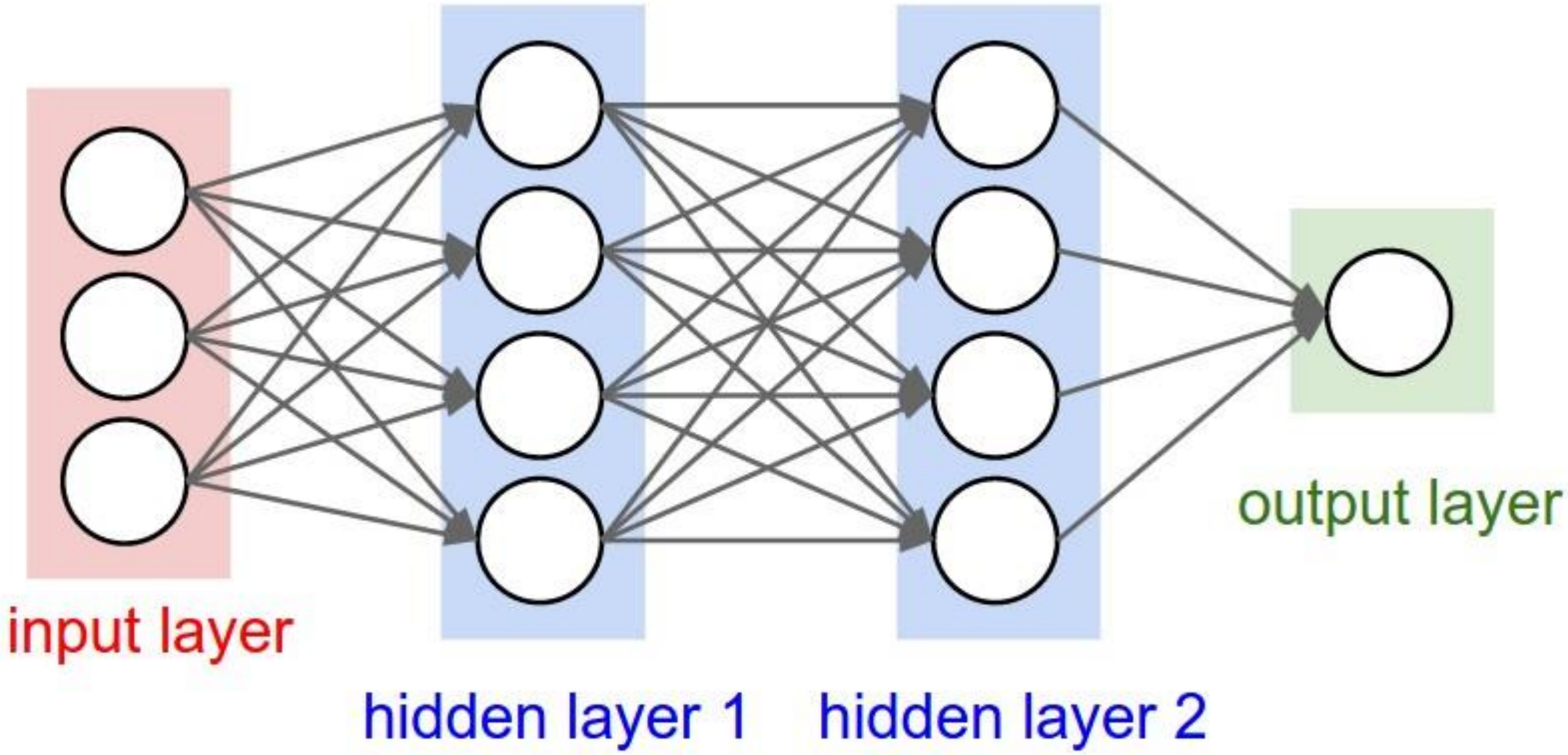


- Can represent nonlinear functions (provided each perceptron has a nonlinearity)

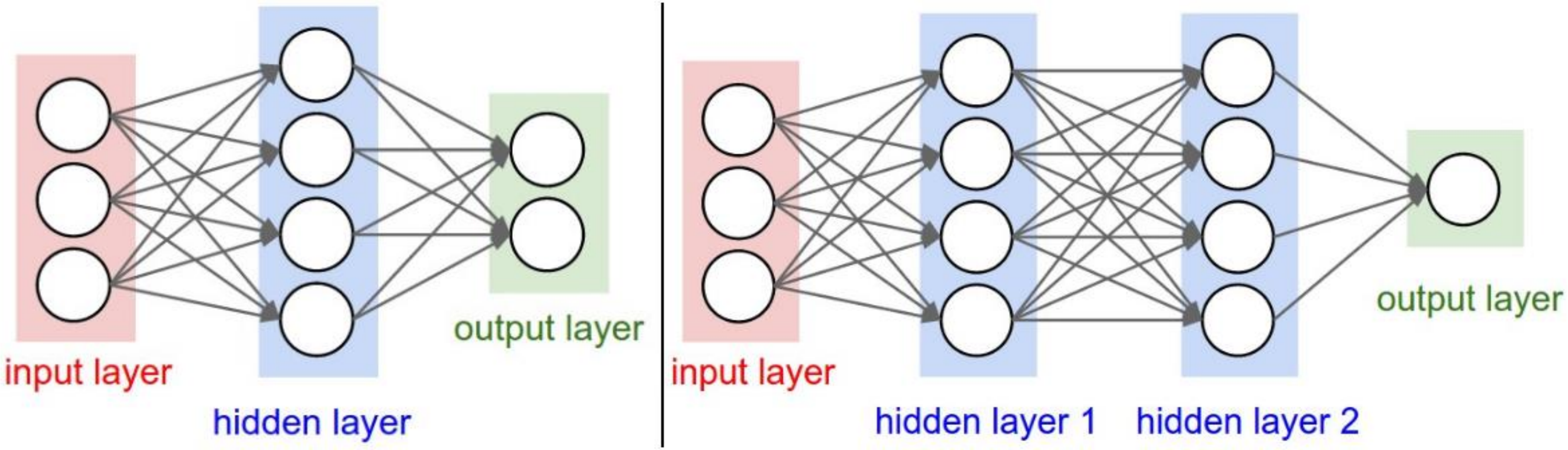
MULTI-LAYER NEURAL NETWORKS

- Beyond a single hidden layer:



Source: http://cs231n.github.io

SIZING NEURAL NETWORKS



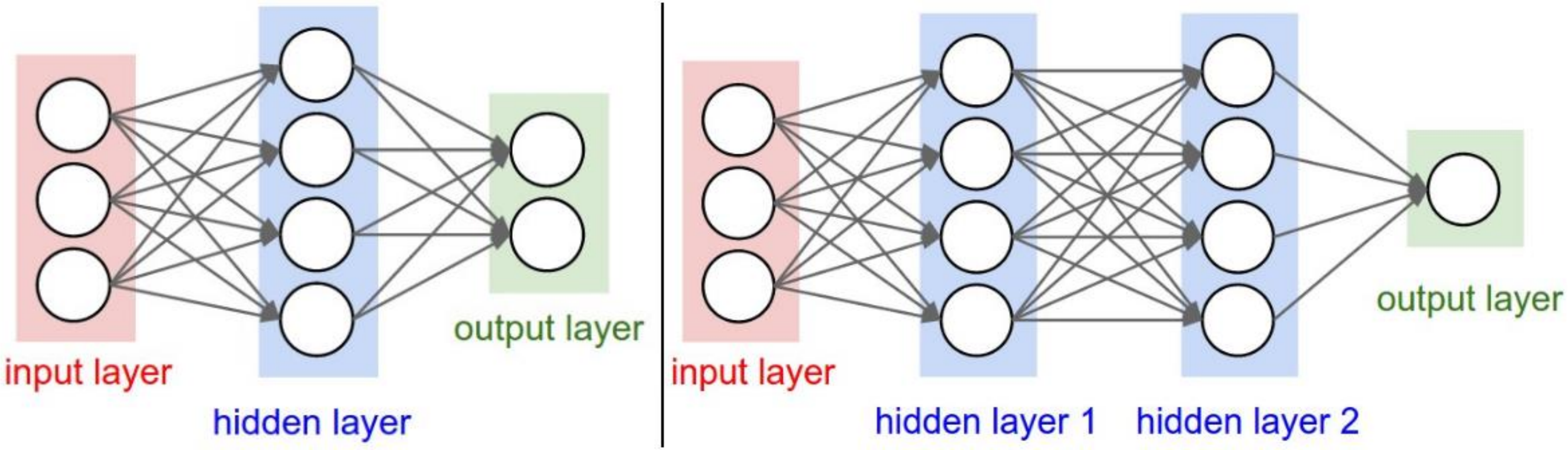
First network (left):

No. of neurons (not counting the inputs):

No. of learnable parameters:

Source: http://cs231n.github.io

SIZING NEURAL NETWORKS



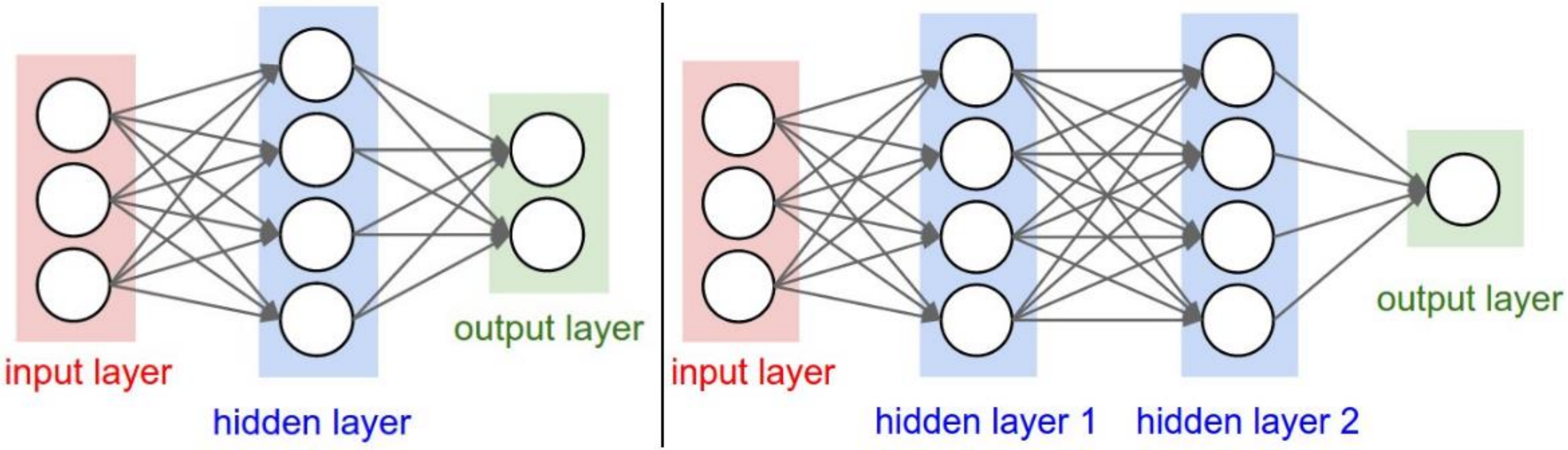
First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters:

Source:
http://cs231n.github.io

SIZING NEURAL NETWORKS



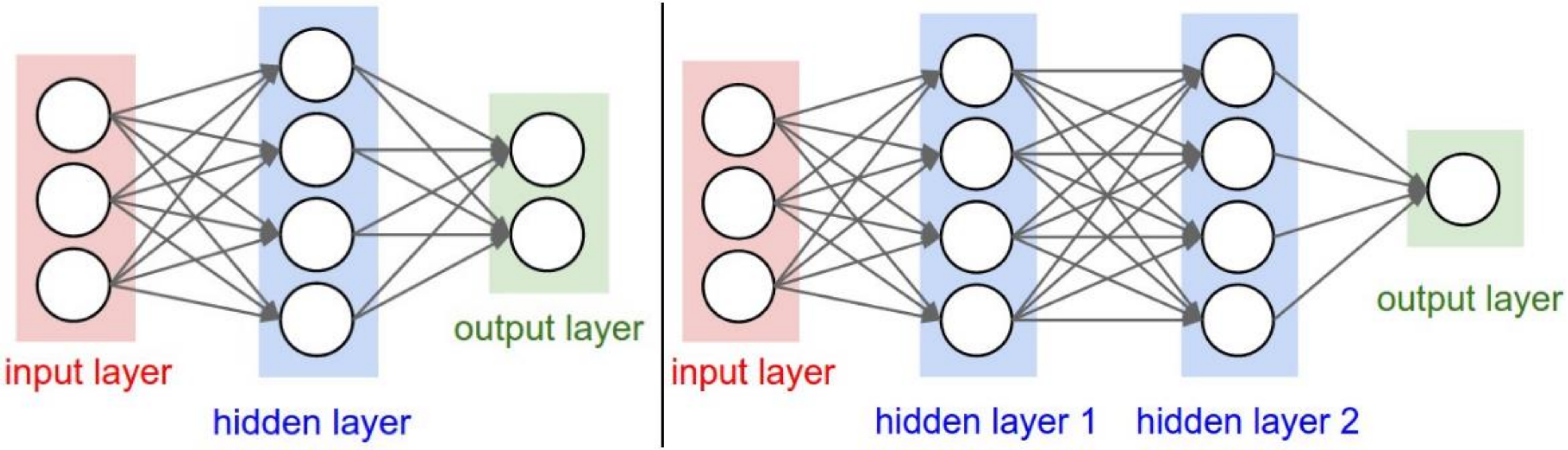
First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters: [3 x 4] + [4 x 2] = 20 weights + 4 + 2 = 6 biases = 26.

Source: http://cs231n.github.io

SIZING NEURAL NETWORKS



First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters: [3 x 4] + [4 x 2] = 20 weights + 4 + 2 = 6 biases = 26.

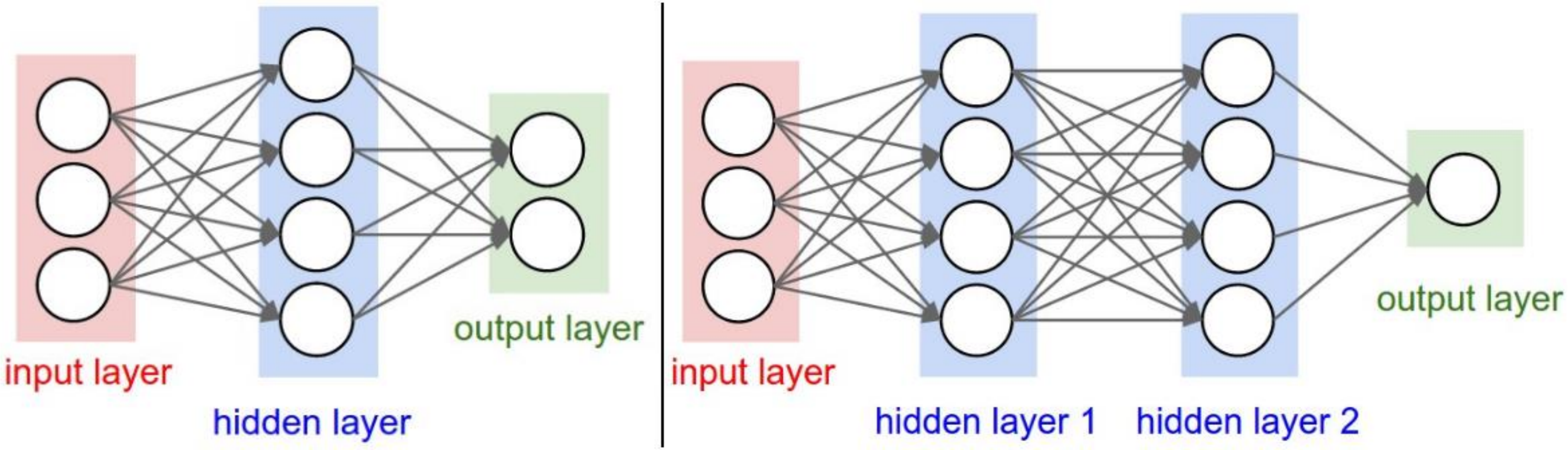
Second network (right):

No. of neurons (not counting the inputs):

No. of learnable parameters:

Source: http://cs231n.github.io

SIZING NEURAL NETWORKS



First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters: [3 x 4] + [4 x 2] = 20 weights + 4 + 2 = 6 biases = 26.

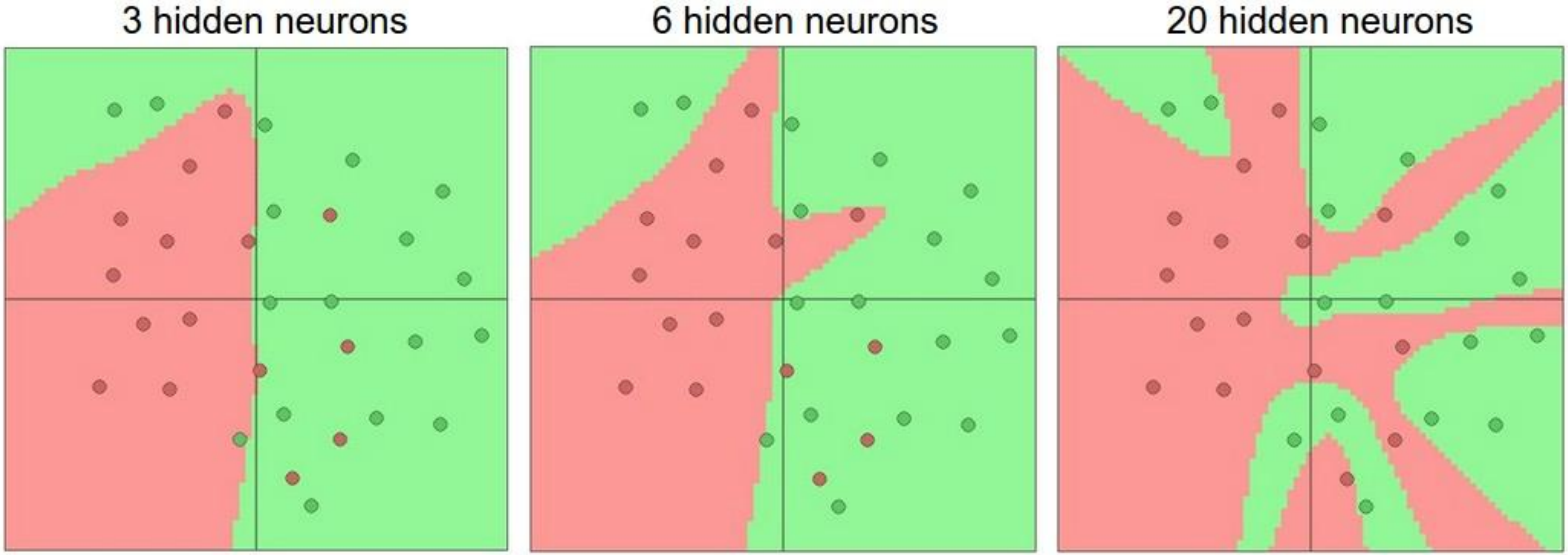
Second network (right):

No. of neurons (not counting the inputs): 4 + 4 + 1 = 9

No. of learnable parameters: [3x4]+[4x4]+[4x1] = 32 weights + 4 + 4 + 1 = 9 biases = 41.

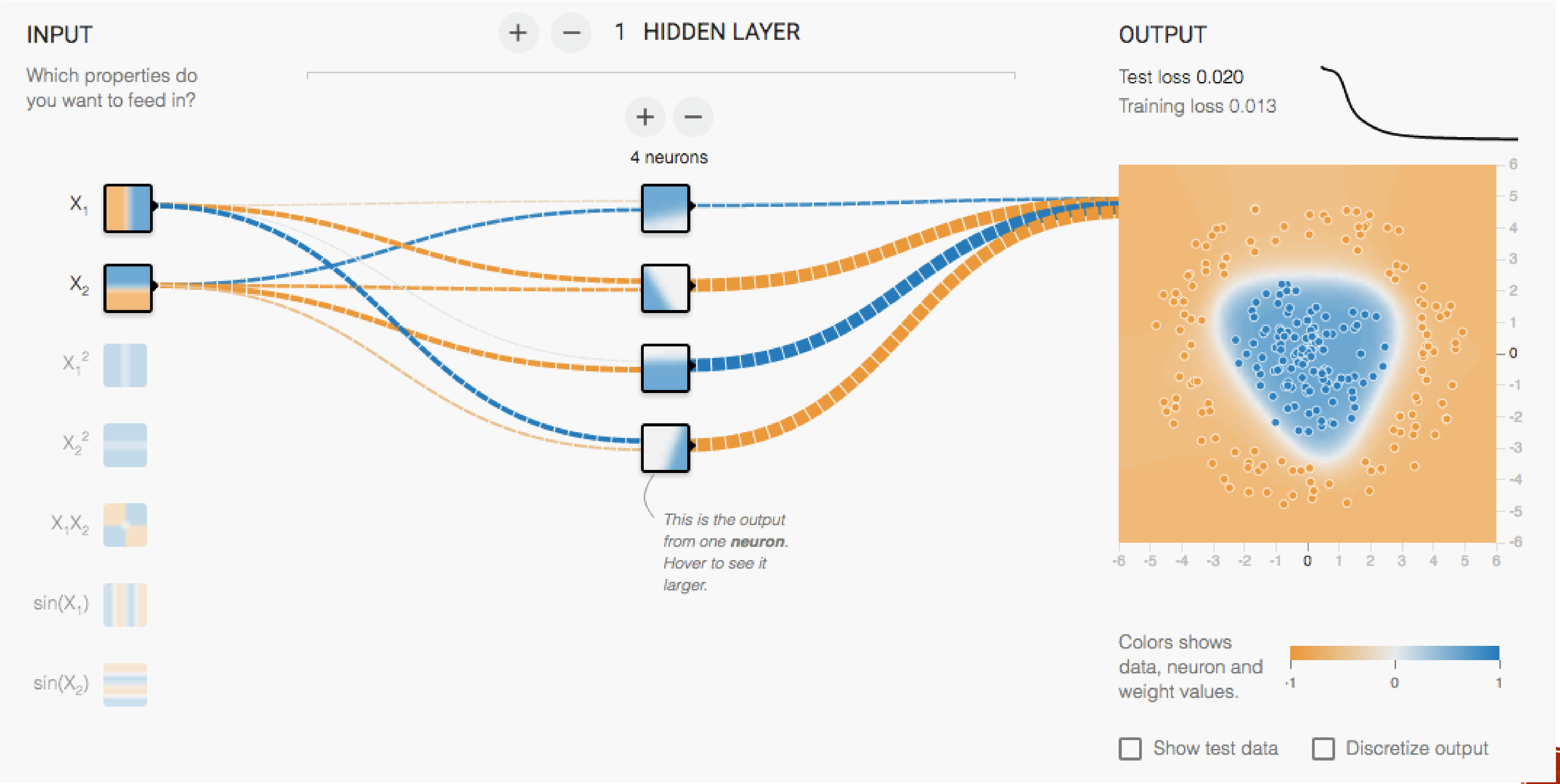
Source: http://cs231n.github.io

MULTI-LAYER NEURAL NETWORKS



Source: http://cs231n.github.io

MULTI-LAYER NETWORK DEMO



http://playground.tensorflow.org/

TRAINING OF MULTI-LAYER NETWORKS

- Find network weights to minimize the error between true and estimated outputs of training examples:

$$E(\mathbf{w}) = \sum_{j=1}^{N} \left(y_j - f_{\mathbf{w}}(\mathbf{x}_j) \right)^2$$

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- **Back-propagation:** gradients are computed in the direction from output to input layers and combined using chain rule

NEURAL NETWORKS: PROS AND CONS

Pros

- Flexible and general function approximation framework
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Pros

- Flexible and general function approximation framework
- Can build extremely powerful models by adding more layers

Cons

- Hard to analyze theoretically (e.g., training is prone to local optima)
- Huge amount of training data, computing power may be required to get good performance
- The space of implementation choices are huge (network architectures, parameters)

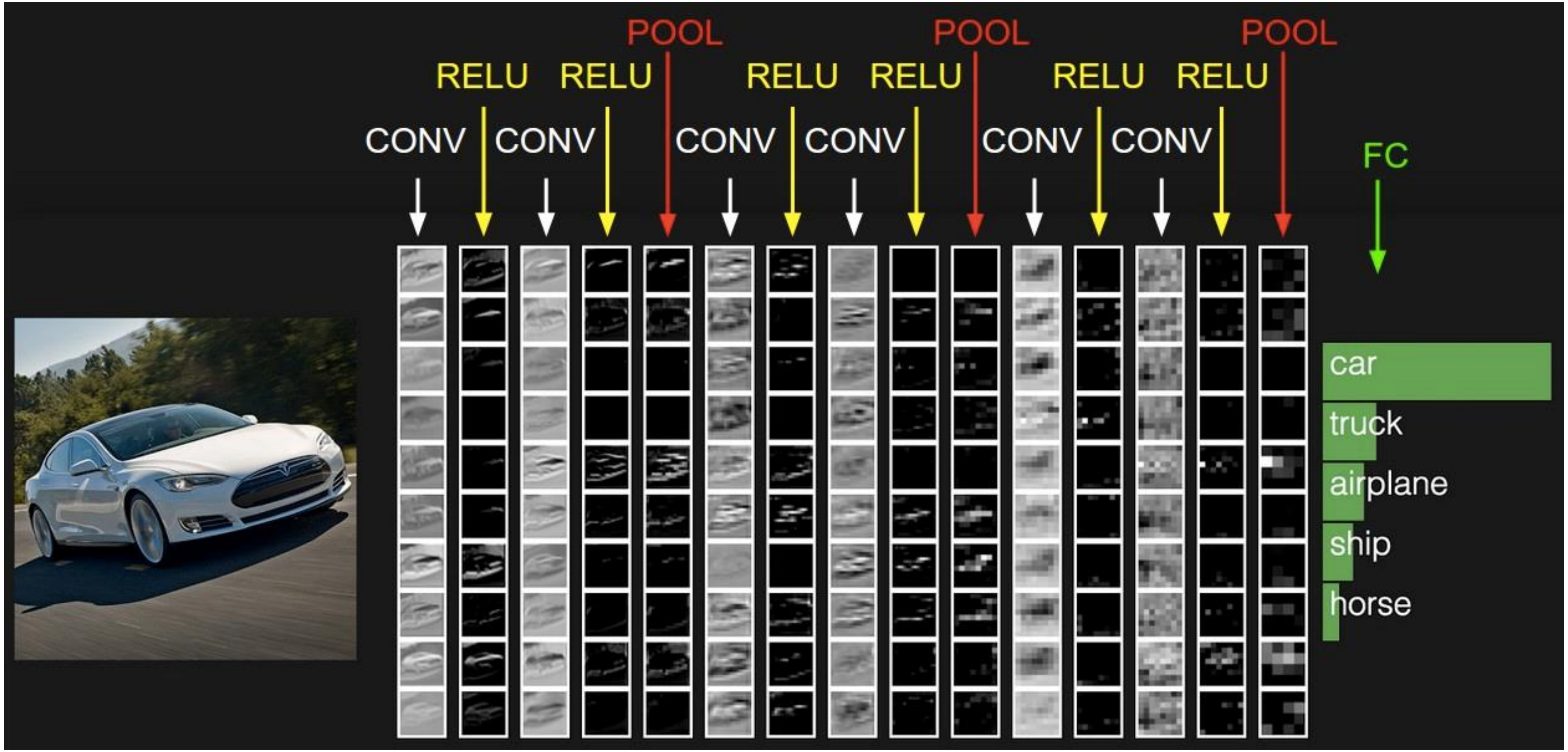
ACKNOWLEDGEMENT

Thanks to the following courses and corresponding researchers for making their teaching/research material online

- Convolutional Neural Networks for Visual Recognition, Stanford University
- Deep Learning, Stanford University
- Introduction to Deep Learning, University of Illinois at Urbana-Champaign
- Introduction to Deep Learning, Carnegie Mellon University
- Natural Language Processing with Deep Learning, Stanford University
- And Many More Publicly Available Resources ……

NEXT LECTURE

Convolutional Neural Networks



Questions?